IB SL MATHEMATICS

CHRISTMAS REVIEW

NAME: _____

DATE: _____

1. (a) On the following diagram, sketch the graphs of $y = e^x$ and $y = \cos x$ for $-2 \le x \le 1$.



(b) The equation $e^x = \cos x$ has a solution between -2 and -1.

Find this solution.

Working:		
	Answers:	
	(a)	
	(b)	
]	(Total 4 marks)

2. Let $f(x) = x^3 - 3x^2 - 24x + 1$.

The tangents to the curve of *f* at the points P and Q are parallel to the *x*-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

- (b) Write down the coordinates of the points where
 - (i) the tangent at P intersects N_2 ;
 - (ii) the tangent at Q intersects N_1 .

(Total 6 marks)

3. Consider the function $h: x \mapsto \frac{x-2}{(x-1)^2}, x \neq 1$.

A sketch of part of the graph of h is given below.



The line (AB) is a vertical asymptote. The point P is a point of inflexion.

(a) Write down the **equation** of the vertical asymptote.

(b) Find h'(x), writing your answer in the form

$$\frac{a-x}{(x-1)^n}$$

where a and n are constants to be determined.

(c) Given that
$$h''(x) = \frac{2x-8}{(x-1)^4}$$
, calculate the coordinates of P.

(3) (Total 8 marks)

(4)

(1)

4. The eye colour of 97 students is recorded in the chart below.

	Brown	Blue	Green
Male	21	16	9
Female	19	19	13

One student is selected at random.

- (a) Write down the probability that the student is a male.
- (b) Write down the probability that the student has green eyes, given that the student is a female.
- (c) Find the probability that the student has green eyes or is male.

(Total 6 marks)

- 5. In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.
 - (a) Find the probability that a student takes physics given that the student takes chemistry.
 - (b) Find the probability that a student takes physics given that the student does **not** take chemistry.
 - (c) State whether the events "taking chemistry" and "taking physics" are mutually exclusive, independent, or neither. Justify your answer.

.....

(Total 6 marks)

- 6. Consider the functions $f: x \mapsto 4(x-1)$ and $g: x \mapsto \frac{6-x}{2}$.
 - (a) Find g^{-1} .
 - (b) Solve the equation $(f \circ g^{-1})(x) = 4$.

Working:	
	Answers:
	(a)

.....

(Total 6 marks)

- 100 80 Frequency 60 40 20 0 20 100 80 40 60 Age in years
- 7. The histogram below represents the ages of 270 people in a village.

Use the histogram to complete the table below. (a)

Age range	Frequency	Mid-interval value
$0 \le age < 20$	40	10
$20 \le age < 40$		
$40 \le age < 60$		
$60 \le age < 80$		
$80 \le age \le 100$		

(b) Hence, calculate an estimate of the mean age.

.....

Christmas Packet 2013

.....

- 8. Let $f(x) = \sin(2x+1), 0 \le x \le \pi$.
 - (a) Sketch the curve of y = f(x) on the grid below.



(b) Find the x-coordinates of the maximum and minimum points of f(x), giving your answers correct to one decimal place.

Working:		
	Answer:	
	(b)	
		(Total 6 marks)

Class	Weight (kg)	Number of boxes	
А	$9.5 \le w < 18.5$	7	
В	$18.5 \le w < 27.5$	12	
C	$27.5 \le w < 36.5$	13	
D $36.5 \le w < 45.5$		10	
Е	$45.5 \le w < 54.5$	8	

9. There are 50 boxes in a factory. Their weights, w kg, are divided into 5 classes, as shown in the following table.

- (a) Show that the estimated mean weight of the boxes is 32 kg.
- (b) There are x boxes in the factory marked "Fragile". They are all in class E. The estimated mean weight of all the other boxes in the factory is 30 kg. Calculate the value of x.
- (c) An additional *y* boxes, all with a weight in class D, are delivered to the factory. The total estimated mean weight of **all** of the boxes in the factory is less than 33 kg. Find the largest possible value of *y*.

(5) (Total 12 marks)

(3)

(4)

10. The equation of a curve may be written in the form y = a(x - p)(x - q). The curve intersects the *x*-axis at A(-2, 0) and B(4, 0). The curve of y = f(x) is shown in the diagram below.



- (a) (i) Write down the value of p and of q.
 - (ii) Given that the point (6, 8) is on the curve, find the value of a.
 - (iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.
- (b) (i) Find $\frac{dy}{dx}$.
 - (ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

(5)

- (c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.
 - (i) Find the equation of *L*.
 - (ii) Find the *x*-coordinate of the point where *L* intersects the curve again.

(6) (Total 15 marks) **11.** The following is the cumulative frequency curve for the time, *t* minutes, spent by 150 people in a store on a particular day.



(a) (i) How many people spent less than 5 minutes in the store?

- (ii) Find the number of people who spent between 5 and 7 minutes in the store.
- (iii) Find the median time spent in the store.

(c)	(i)	On your answe	er sheet, copy and	complete the fo	ollowing frequency table.
(-)	(-)		·- ····· · · · · · · · · · · · · · · ·		

t (minutes)	$0 \le t < 2$	$2 \le t < 4$	$4 \le t < 6$	$6 \le t < 8$	$8 \le t < 10$	$10 \le t < 12$
Frequency	10	23				15

(ii) Hence, calculate an estimate for the mean time spent in the store.

(5) (Total 14 marks)

(2)

12. Let
$$g(x) = x^4 - 2x^3 + x^2 - 2$$
.

(a) Solve
$$g(x) = 0$$
.

Let $f(x) = \frac{2x^3}{g(x)} + 1$. A part of the graph of f(x) is shown below.



- (b) The graph has vertical asymptotes with equations x = a and x = b where a < b. Write down the values of
 - (i) *a*;
 - (ii) *b*.

(2)

- (c) The graph has a horizontal asymptote with equation y = 1. Explain why the value of f(x) approaches 1 as x becomes very large.
- (d) The graph intersects the *x*-axis at the points A and B. Write down the **exact** value of the *x*-coordinate at
 - (i) A;
 - (ii) B.

(2)

(2)

(e) The curve intersects the *y*-axis at C. Use the graph to explain why the values of f'(x) and f''(x) are zero at C.

(2) (Total 10 marks) **13.** The diagram below shows a quadrilateral ABCD. AB = 4, AD = 8, CD = 12, $B\hat{C}D = 25^{\circ}$, $B\hat{A}D = \theta$.



(a) Use the cosine rule to show that $BD = 4\sqrt{5-4\cos\theta}$.

Let $\theta = 40^{\circ}$.

- (b) (i) Find the value of sin \hat{CBD} .
 - (ii) Find the two possible values for the size of $\ C\hat{B}D$.
 - (iii) Given that \hat{CBD} is an acute angle, find the perimeter of ABCD.
- (c) Find the area of triangle ABD.

(2) (Total 16 marks)

(2)

(12)

Christmas Packet 2013

14. (a) Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

- (i) the value of *x* giving the maximum value of *y*;
- (ii) this maximum value of *y*.

The triangle XYZ has XZ = 6, YZ = x, XY = z as shown below. The perimeter of triangle XYZ is 16.



- (b) (i) Express z in terms of x.
 - (ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$.

(iii) Hence, show that
$$\cos Z = \frac{5x-16}{3x}$$
. (7)

Let the area of triangle XYZ be A.

(c) Show that
$$A^2 = 9x^2 \sin^2 Z$$
.

- (d) Hence, show that $A^2 = -16x^2 + 160x 256$.
- (e) (i) Hence, write down the maximum area for triangle XYZ.
 - (ii) What type of triangle is the triangle with maximum area?

(3) (Total 20 marks)

(4)

(4)

(2)