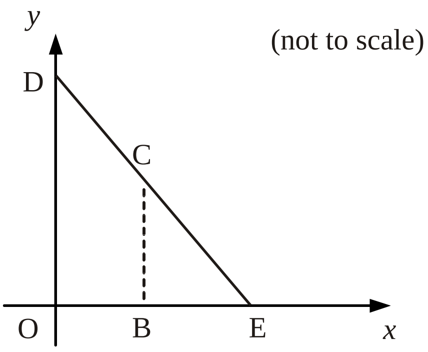
Functions Questions.

1. On the coordinate axes below, D is a point on the *y*-axis and E is a point on the *x*-axis.

O is the origin. The equation of the line DE is $y + \frac{1}{2}x = 4$.



(a) Write down the coordinates of point E.

(2)

C is a point on the line DE. B is a point on the *x*-axis such that BC is parallel to the *y*-axis. The *x*-coordinate of C is *t*.

(b) Show that the y-coordinate of C is $4 - \frac{1}{2}t$.

(2)

OBCD is a trapezium. The y-coordinate of point D is 4.

(c) Show that the area of OBCD is $4t - \frac{1}{4}t^2$.

(3)

(d) The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information.

(1)

(e) (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d).

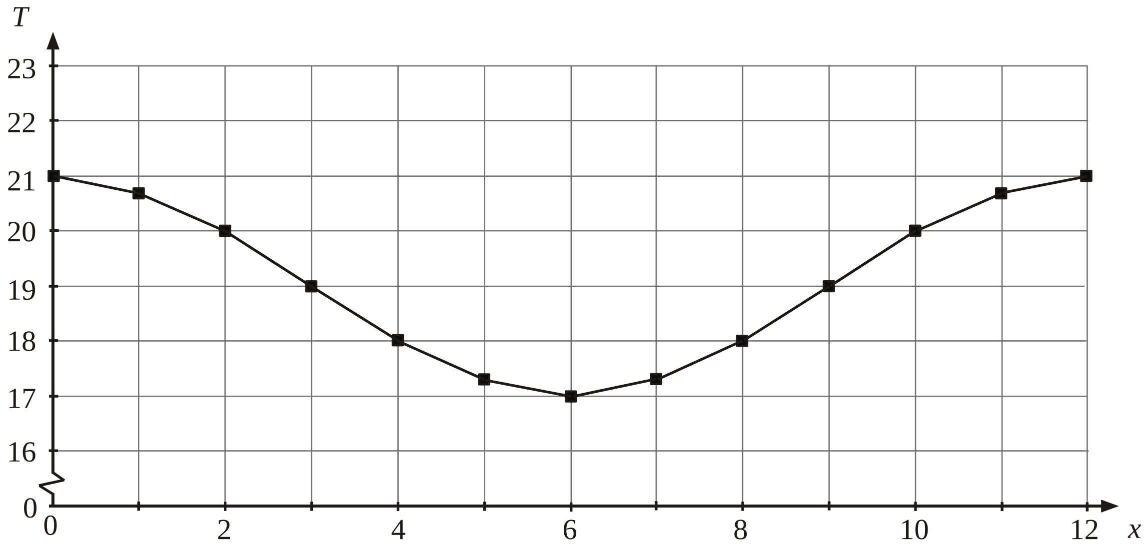
(ii) Hence find the correct value for *t*. Give a reason for your answer.

(4)
(Total 12 marks)

2. The graph below represents the temperature (T° Celsius) in Washington measured at midday during a period of thirteen consecutive days starting at Day 0.
These points also lie on the graph of the function

$$T(x) = a + b\cos(cx°),\ 0 \le x \le 12,$$

where a, b and $c \in \mathbb{R}$.



(a) Find the value of

(i) a;

(ii) b.

(2)

(b) Show that $c = 30$.

(1)

(c) Using the graph, or otherwise, write down the part of the domain for which the midday temperature is less than 18.5°.

(2)
(Total 5 marks)

3. (a) Sketch the graph of the function $f(x) = \dfrac{2x+3}{x+4}$, for $-10 \leq x \leq 10$. Indicating clearly the axis intercepts and any asymptotes.

(6)

(b) Write down the equation of the vertical asymptote.

(2)

(c) On the same diagram sketch the graph of $g(x) = x + 0.5$.

(2)

(d) Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**.

(3)

(e) Write down the gradient of the line $g(x) = x + 0.5$.

(1)

(f) The line L passes through the point with coordinates $(-2, -3)$ and is perpendicular to the line $g(x)$. Find the equation of L.

(3)
(Total 17 marks)

4. (a) Factorize the expression $x^2 - 3x - 10$.

(2)

(b) A function is defined as $f(x) = 1 + x^3$ for $x \in \mathbb{Z}, -3 \leq x \leq 3$.

(i) List the elements of the domain of $f(x)$.

(ii) Write down the range of $f(x)$.

(4)

Working:

Answers:

(a) ....

(b) (i)..

(ii)..

(Total 6 marks)

5. The depth, in metres, of water in a harbour is given by the function $d = 4 \sin (0.5t°) + 7$, where t is in minutes, $0 \le t < 1440$.

(a) Write down the amplitude of d.

(1)

(b) Find the maximum value of d.

(1)

(c) Find the period of d. Give your answer in **hours**.

(2)

On Tuesday, the minimum value of d occurs at 14:00.

(d) Find when the next **maximum** value of d occurs.

(2)

Working:

Answers:

(a) ....

(b) ....

(c) ....

(d) ....

(Total 6 marks)

6. In an experiment it is found that a culture of bacteria triples in number every four hours. There are 200 bacteria at the start of the experiment.

Hours	0	4	8	12	16
No. of bacteria	200	600	a	5400	16200

(a) Find the value of a.

(1)

(b) Calculate how many bacteria there will be after one day.

(2)

(c) Find how long it will take for there to be two million bacteria.

(3)

Working:

Answers:

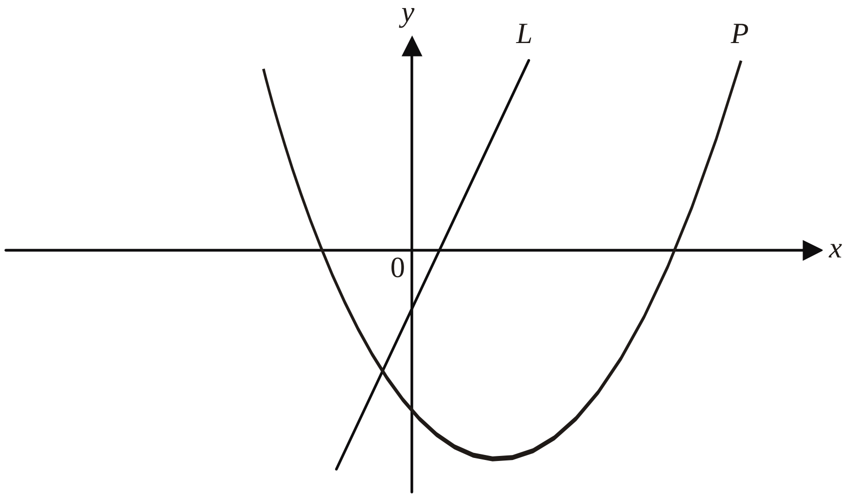
(a) ....

(b) ....

(c) ....

(Total 6 marks)

7. The diagram below shows the graph of a line L passing through (1, 1) and (2, 3) and the graph P of the function $f(x) = x^2 - 3x - 4$



(a) Find the gradient of the line L.

(2)

(b) Differentiate $f(x)$.

(2)

(c) Find the coordinates of the point where the tangent to P is parallel to the line L.

(3)

(d) Find the coordinates of the point where the tangent to P is perpendicular to the line L.

(4)

(e) Find

(i) the gradient of the tangent to P at the point with coordinates $(2, -6)$;

(ii) the equation of the tangent to P at this point.

(3)

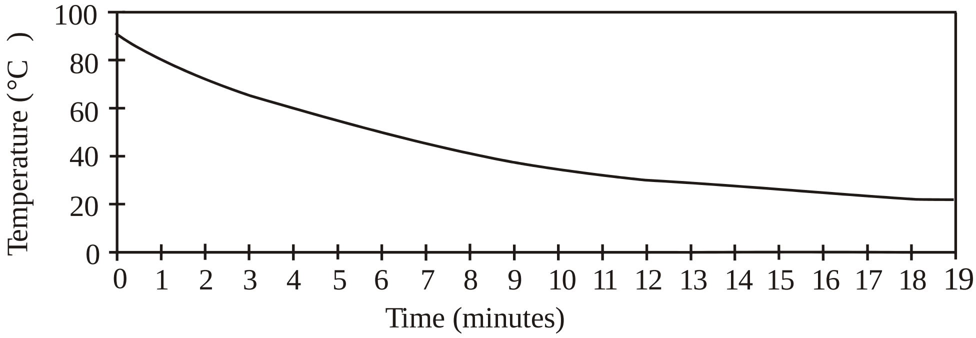
(f) State the equation of the axis of symmetry of P.

(1)

(e) Find the coordinates of the vertex of P and state the gradient of the curve at this point.

(3)
(Total 18 marks)

8. The following graph shows the temperature in degrees Celsius of Robert's cup of coffee, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the coffee out.



(a) Find the initial temperature of the coffee.

(1)

(b) Write down the equation of the horizontal asymptote.

(1)

(c) Find the room temperature.

(1)

(d) Find the temperature of the coffee after 10 minutes.

(1)

If the coffee is not hot enough it is reheated in a microwave oven. The liquid increases in temperature according to the formula

$$T = A \times 2^{1.5t}$$

where T is the final temperature of the liquid, A is the initial temperature of coffee in the microwave and t is the time in minutes after switching the microwave on.

(e) Find the temperature of Robert's coffee after being heated in the microwave for **30 seconds** after it has reached the temperature in part (d).

(3)

(f) Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C.

(4)
(Total 11 marks)

9. *It is **not** necessary to use graph paper for this question.*

(a) Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from -2 to 4, giving the intercepts with both axes.

(3)

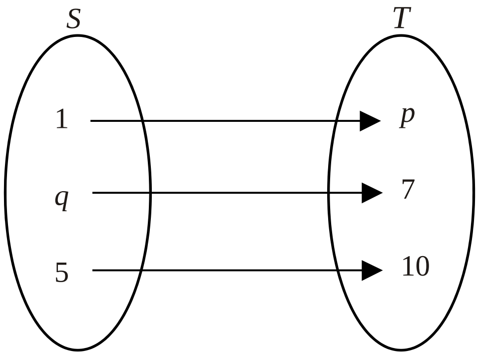
(b) On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve.

(3)

(c) Find the value of the gradient of the curve where $x = 1.7$.

(2)
(Total 8 marks)

10. (a) $f : x \to 3x - 5$ is a mapping from the set S to the set T as shown below.



Find the values of p and q.

(2)

(b) A function g is such that $g(x) = \dfrac{2}{(x-2)^2}$.

(i) State the domain of the function $g(x)$.

(2)

(ii) State the range of the function $g(x)$.

(1)

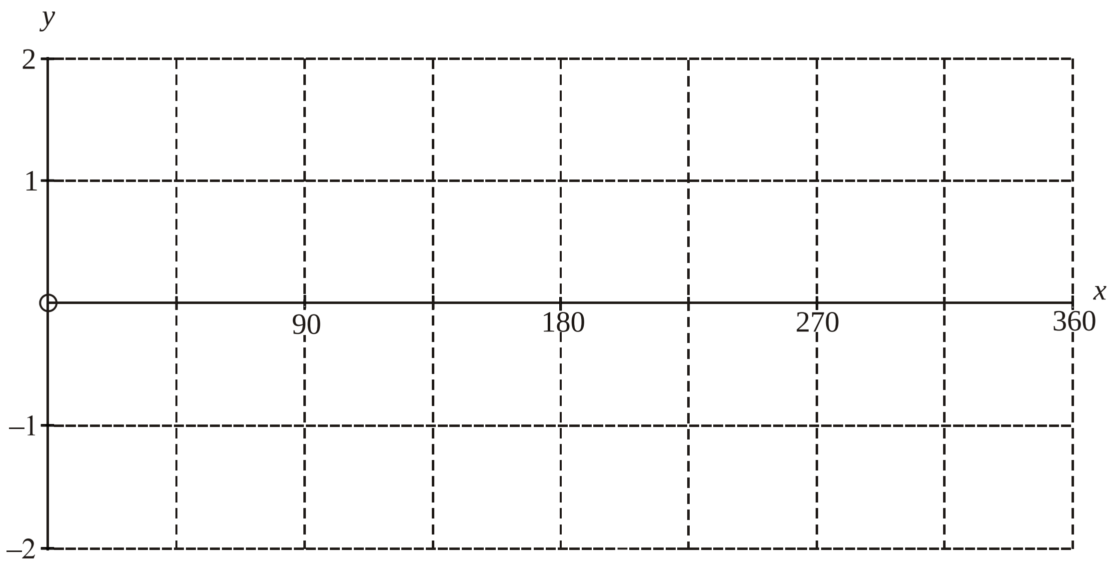
(iii) Write down the equation of the vertical asymptote.

(1)

(Total 6 marks)

11. (a) Sketch the graph of the function $y = 1 + \frac{\sin(2x)}{2}$ for $0° \le x \le 360°$ on the axes below.

(4)



(b) Write down the period of the function.

(1)

(c) Write down the amplitude of the function.

(1)

Working:

Answers:

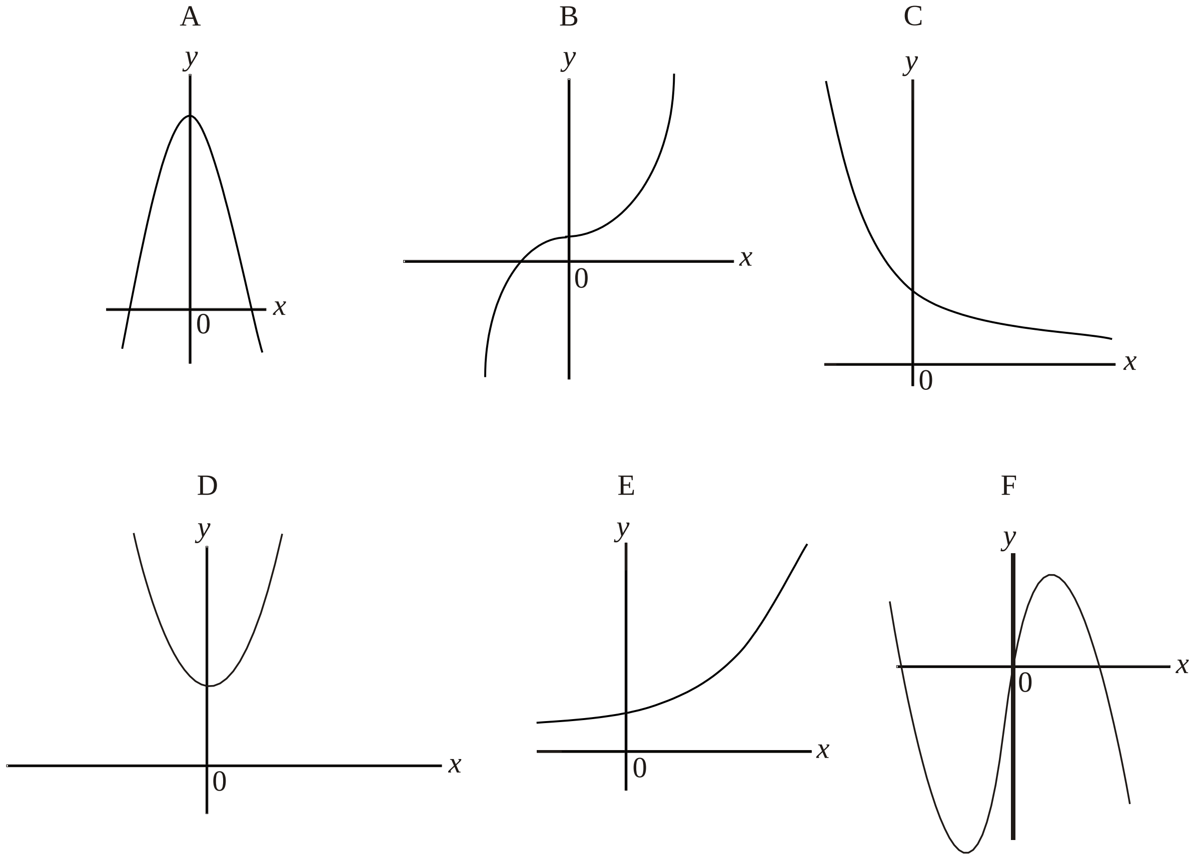
(b) ....

(c) ....

(Total 6 marks)

12. The following curves are sketches of the graphs of the functions given below, but in a different order. Using your graphic display calculator, match the equations to the curves, writing your answers in the table below.

(the diagrams are not to scale)



	Function	Graph label
(i)	$y = x^3 + 1$	
(ii)	$y = x^2 + 3$	
(iii)	$y = 4 - x^2$	
(iv)	$y = 2x + 1$	
(v)	$y = 3^{-x} + 1$	
(vi)	$y = 8x - 2x^2 - x^3$	

(Total 6 marks)