SECTION 1: STATISTICS

1. At a conference of 100 mathematicians there are 72 men and 28 women. The men have a mean height of 1.79 m and the women have a mean height of 1.62 m. Find the mean height of the 100 mathematicians.

(Total 4 marks)

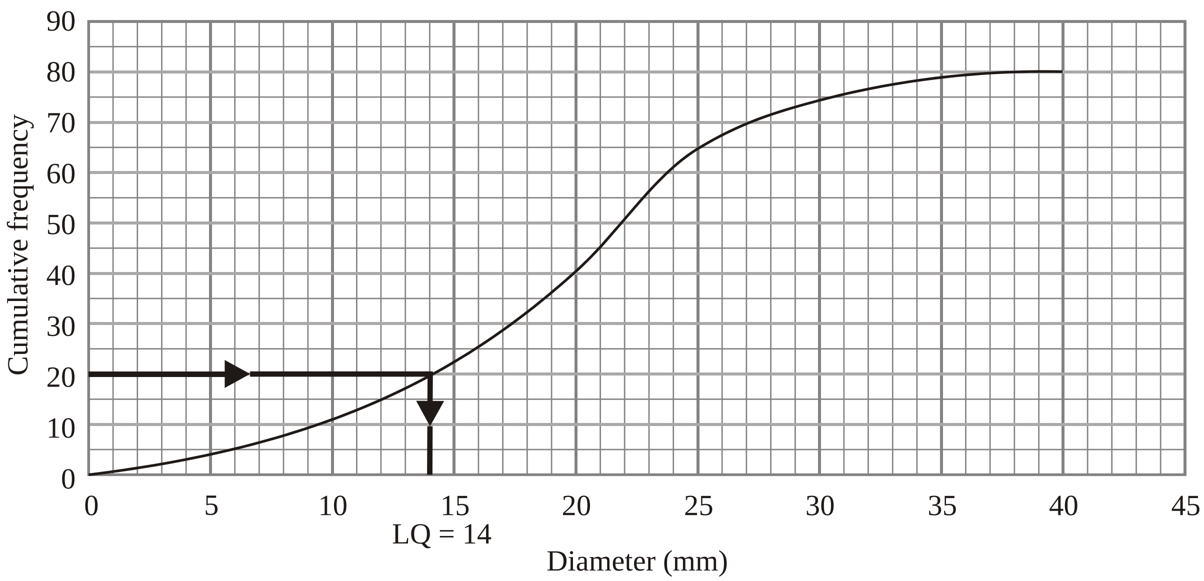
2. The table shows the scores of competitors in a competition.

Score	10	20	30	40	50
Number of competitors with this score	1	2	5	k	3

The mean score is 34. Find the value of k.

(Total 4 marks)

3. A student measured the diameters of 80 snail shells. His results are shown in the following cumulative frequency graph. The lower quartile (LQ) is 14 mm and is marked clearly on the graph.



(a) On the graph, mark clearly in the same way and write down the value of

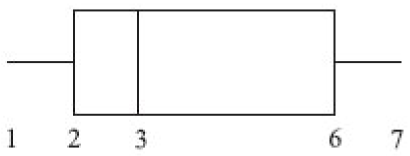
(i) the median;

(ii) the upper quartile.

(b) Write down the interquartile range.

(Total 6 marks)

4. The box and whisker diagram shown below represents the marks received by 32 students.



(a) Write down the value of the median mark.

(b) Write down the value of the upper quartile.

(c) Estimate the number of students who received a mark greater than 6.

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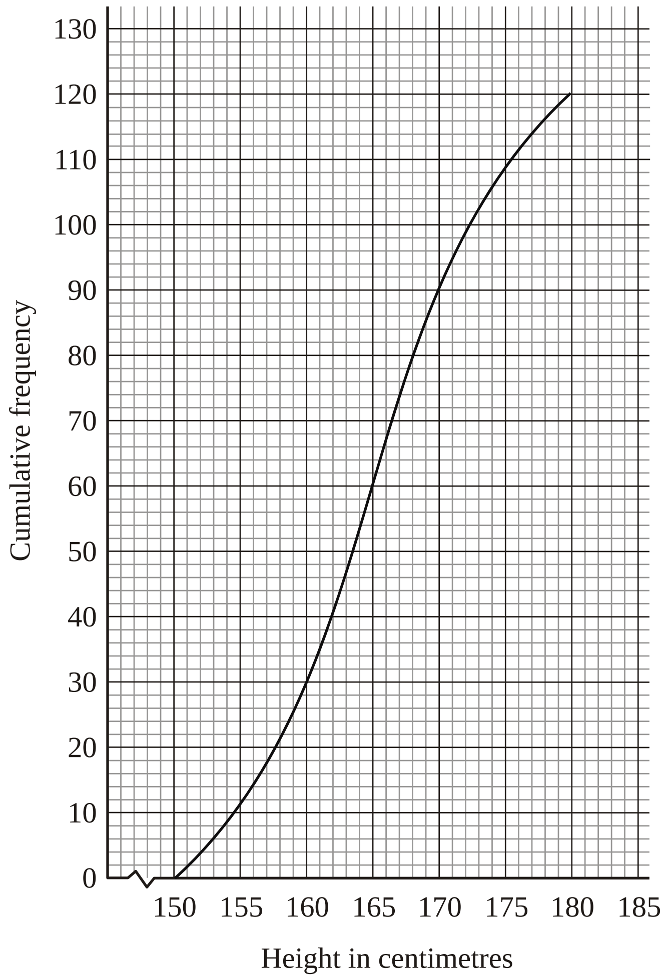
(Total 6 marks)

5. The cumulative frequency graph below shows the heights of 120 girls in a school.



(a) Using the graph

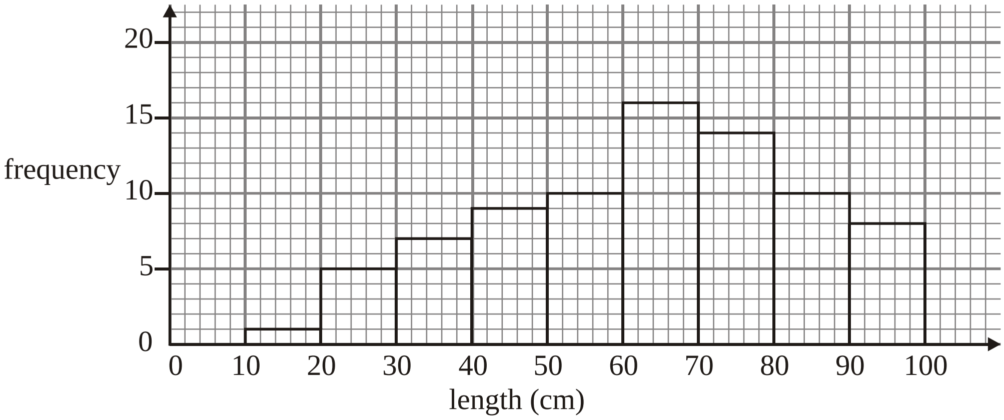
(i) write down the median;

(ii) find the interquartile range.

(b) Given that 60% of the girls are taller than *a* cm, find the value of *a*.

(Total 6 marks)

6. The following diagram represents the lengths, in cm, of 80 plants grown in a laboratory.



(a) How many plants have lengths in cm between

(i) 50 and 60?

(ii) 70 and 90?

(2)

(b) Calculate estimates for the mean and the standard deviation of the lengths of the plants.

(4)

(c) Explain what feature of the diagram suggests that the median is different from the mean.

(1)

(d) The following is an extract from the cumulative frequency table.

length in cm less than	cumulative frequency
.	.
50	22
60	32
70	48
80	62
.	.

Use the information in the table to estimate the median. Give your answer to **two** significant figures.

(3)
(Total 10 marks)

SECTION 2: PROBABILITY

7. In a survey, 100 students were asked “do you prefer to watch television or play sport?” Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

By completing this table or otherwise, find the probability that

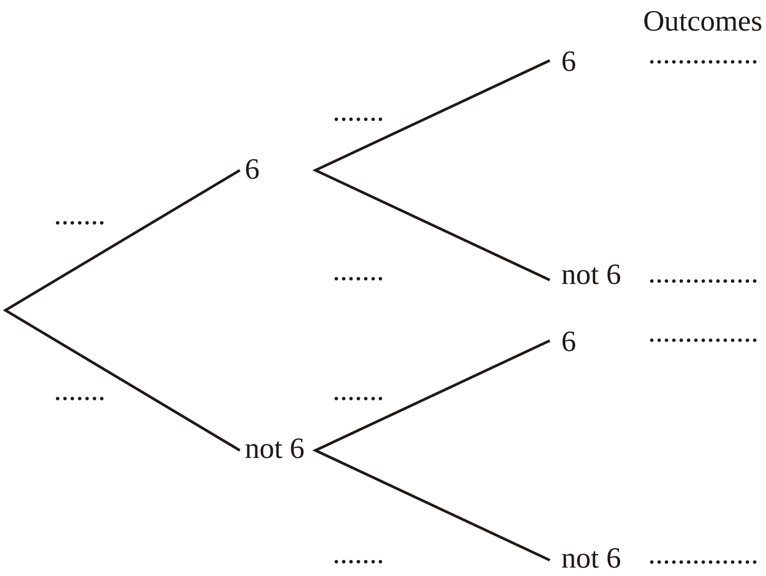
(a) a student selected at random prefers to watch television;

(b) a student prefers to watch television, given that the student is a boy.

(Total 4 marks)

8. Two ordinary, 6-sided dice are rolled and the total score is noted.

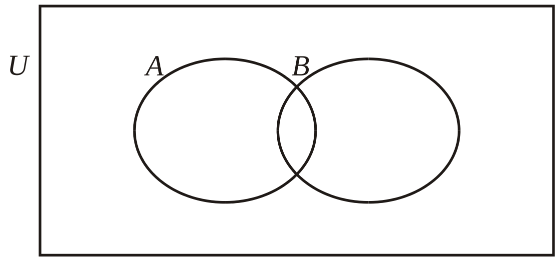
(a) Complete the tree diagram by entering probabilities and listing outcomes.



(b) Find the probability of getting one or more sixes.

(Total 4 marks)

9. The following Venn diagram shows a sample space U and events A and B.



$n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.

(a) On the diagram, shade the region $(A \cup B)'$.

(b) Find

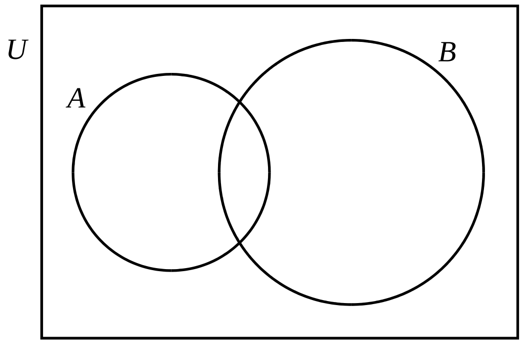
(i) $n(A \cap B)$;

(ii) $P(A \cap B)$.

(c) Explain why events A and B are not mutually exclusive.

(Total 4 marks)

10. The following Venn diagram shows the universal set U and the sets A and B.



(a) Shade the area in the diagram which represents the set $B \cap A'$.

$n(U) = 100$, $n(A) = 30$, $n(B) = 50$, $n(A \cup B) = 65$.

(b) Find $n(B \cap A')$.

(c) An element is selected at random from U. What is the probability that this element is in $B \cap A'$?

(Total 4 marks)

11. A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

(a) The first two apples are green. What is the probability that the third apple is red?

(b) What is the probability that exactly two of the three apples are red?

(Total 6 marks)

SECTION 3: SEQUENCES

13. The first four terms of a sequence are 18, 54, 162, 486.

(a) Use all four terms to show that this is a geometric sequence.

(2)

(b) (i) Find an expression for the n^{th} term of this geometric sequence.

(ii) If the n^{th} term of the sequence is 1062 882, find the value of n.

....

(4)
(Total 6 marks)

14. One of the terms of the expansion of $(x + 2y)^{10}$ is $ax^8 y^2$. Find the value of a.

(Total 6 marks)

15. Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in x^{12}.

(Total 6 marks)

16. Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q.

(a) $\log_a 10$

(b) $\log_a 8$

(c) $\log_a 2.5$

....

(Total 6 marks)

17. Consider the infinite geometric sequence 25, 5, 1, 0.2, … .

(a) Find the common ratio.

(b) Find

(i) the 10^{th} term;

(ii) an expression for the n^{th} term.

(c) Find the sum of the infinite sequence.

....

(Total 6 marks)

18. Consider the infinite geometric series 405 + 270 + 180 +....

(a) For this series, find the common ratio, giving your answer as a fraction in its simplest form.

(b) Find the fifteenth term of this series.

(c) Find the **exact** value of the sum of the infinite series.

....

(Total 6 marks)

19. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

....

(Total 6 marks)

20. (a) Consider the geometric sequence −3, 6, −12, 24, ….

(i) Write down the common ratio.

(ii) Find the 15th term.

Consider the sequence $x - 3$, $x + 1$, $2x + 8$, ….

(3)

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio.

(2)

(c) Find the other value of x for which the sequence is geometric.

(4)

(d) For this value of x, find

(i) the common ratio;

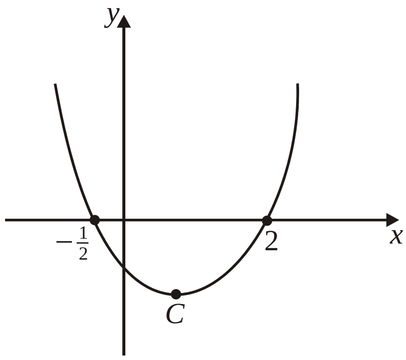
(ii) the sum of the infinite sequence.

(3)
(Total 12 marks)

SECTION 4: FUNCTIONS

21. The diagram represents the graph of the function

$$f : x \mapsto (x - p)(x - q).$$



(a) Write down the values of p and q.

(b) The function has a minimum value at the point C. Find the x-coordinate of C.

(Total 4 marks)

22. Two functions f and g are defined as follows:

$f(x) = \cos x, \quad 0 \le x \le 2\pi;$

$g(x) = 2x + 1, \quad x \in \mathbb{R}.$

Solve the equation $(g \circ f)(x) = 0$.

(Total 4 marks)

23. Two functions f, g are defined as follows:

$f : x \to 3x + 5$
$g : x \to 2(1 - x)$

Find

(a) $f^{-1}(2)$;

(b) $(g \circ f)(-4)$.

(Total 4 marks)

24. The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x.
Find the value of k.

(Total 4 marks)

25. $f(x) = 4\sin\left(3x + \frac{\pi}{2}\right)$.

For what values of k will the equation $f(x) = k$ have no solutions?

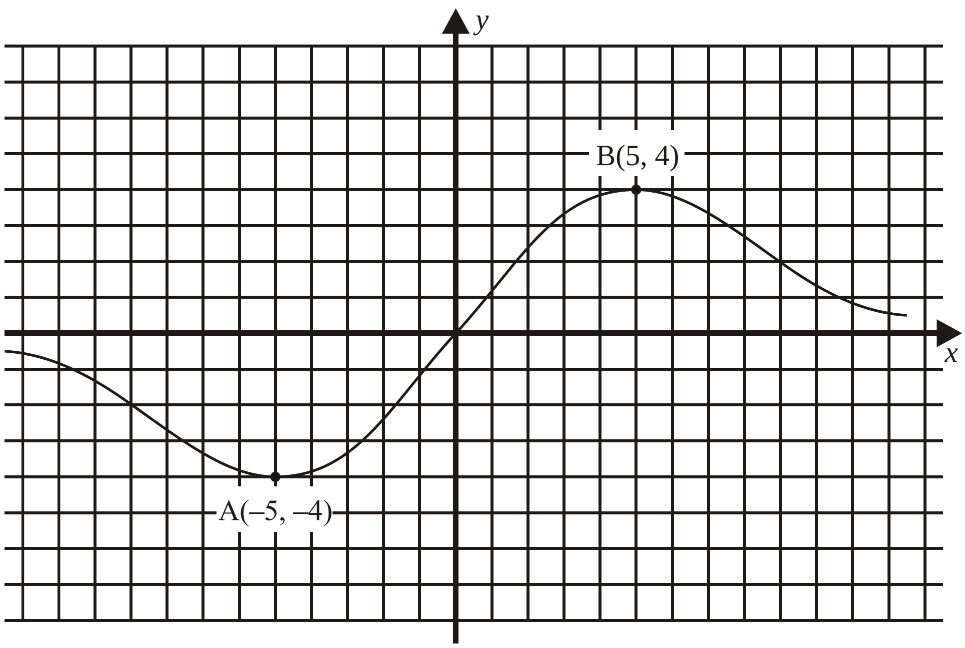
(Total 4 marks)

26. (a) Express $f(x) = x^2 - 6x + 14$ in the form $f(x) = (x - h)^2 + k$, where h and k are to be determined.

(b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation $y - x^2 - 6x + 14$.

(Total 4 marks)

27. The diagram shows the graph of $y = f(x)$, with the x-axis as an asymptote.

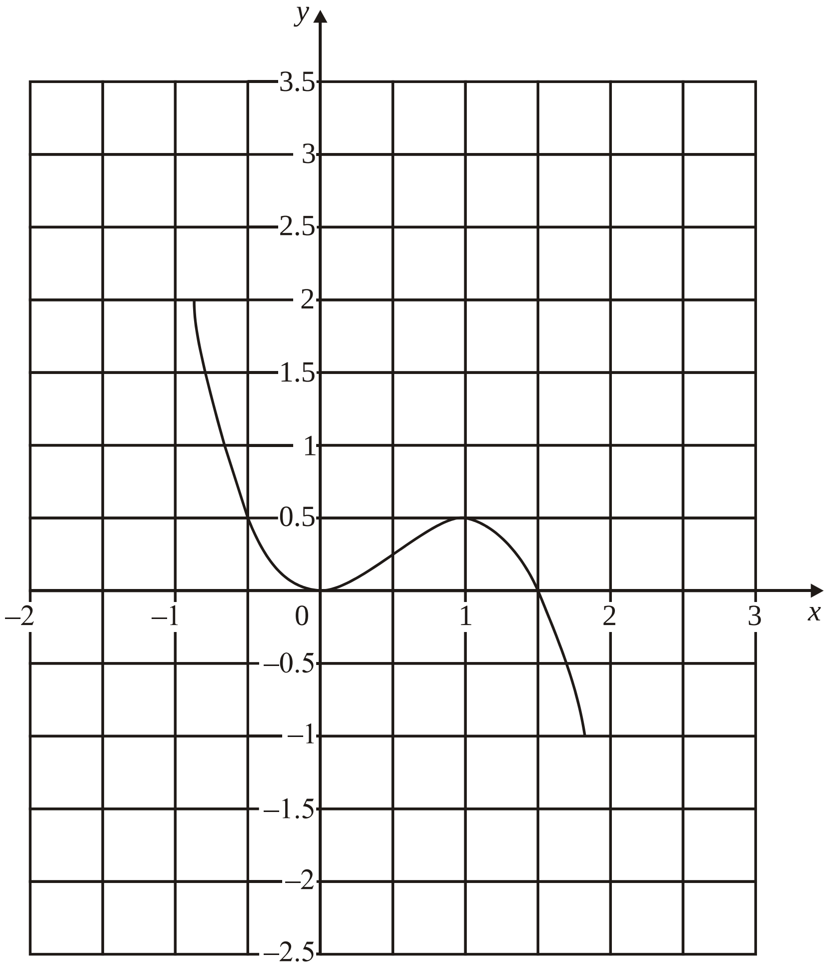


(a) On the same axes, draw the graph of $y = f(x + 2) - 3$, indicating the coordinates of the images of the points A and B.

(b) Write down the equation of the asymptote to the graph of $y = f(x + 2) - 3$.

(Total 4 marks)

28. The following diagram shows the graph of $y = f(x)$. It has minimum and maximum points at $(0, 0)$ and $(1, \frac{1}{2})$.



(a) On the same diagram, draw the graph of $y = f(x-1)+\frac{3}{2}$.

(b) What are the coordinates of the minimum and maximum points of $y = f(x-1)+\frac{3}{2}$?

(Total 4 marks)

29. Let $f(x) = 2^x$, and $g(x) = \dfrac{x}{x-2}$, $(x \neq 2)$.

Find

(a) $(g \circ f)(3)$;

(b) $g^{-1}(5)$.

(Total 6 marks)

30. Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x - p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b) Find the minimum value of $f(x)$.

(Total 6 marks)

31. The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

(Total 6 marks)

32. Consider the function $f(x) = \dfrac{16}{x-10} + 8$, $x \neq 10$.

(a) Write down the **equation** of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find the

(i) y-intercept;

(ii) x-intercept.

(2)

(c) Sketch the graph of f, clearly showing the above information.

(4)

(d) Let $g(x) = \dfrac{16}{x}$, $x \neq 0$.

The graph of g is transformed into the graph of f using two transformations.

The first is a translation with vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$. Give a full geometric description of the second transformation.

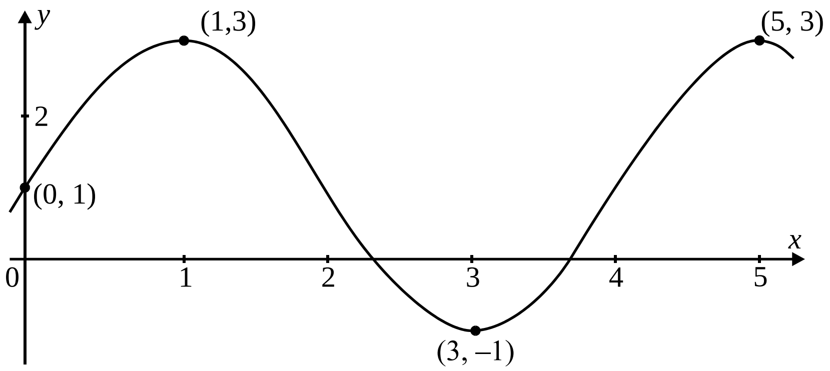
(2)
(Total 10 marks)

SECTION 5: TRIGONOMETRY

33. The diagram shows the graph of the function f given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \le x \le 5$, where A and B are constants, and x is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a) Write down the values of $f(1)$ and $f(5)$.

(2)

(b) Show that the period of f is 4.

(2)

The point (3, –1) is a minimum point of the graph.

(c) Show that $A = 2$, and find the value of B.

(5)

(d) Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(4)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \le x \le 5$.

(e) Find

(i) the point where this tangent meets the curve;

(ii) the value of k.

(6)

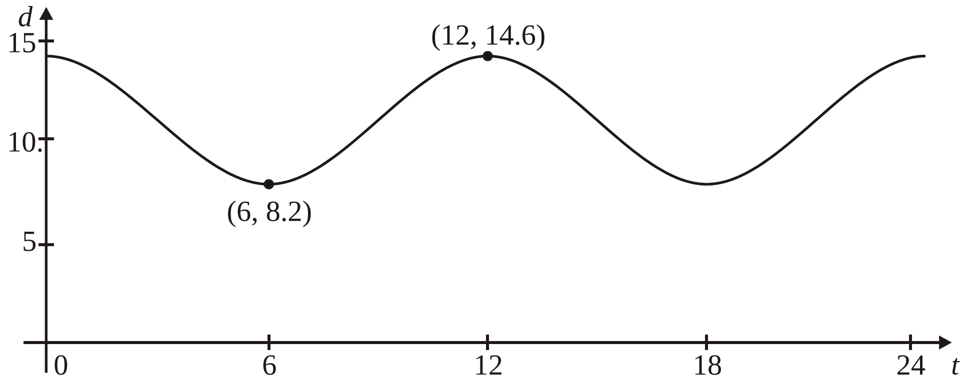
(f) Solve the equation $f(x) = 2$ for $0 \le x \le 5$.

(5)
(Total 24 marks)

34. A formula for the depth d metres of water in a harbour at a time t hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where P and Q are positive constants. In the following graph the point (6, 8.2) is a minimum point and the point (12, 14.6) is a maximum point.



(a) Find the value of

(i) Q;

(ii) P.

(3)

(b) Find the **first** time in the 24-hour period when the depth of the water is 10 metres.

(3)

(c) (i) Use the symmetry of the graph to find the **next** time when the depth of the water is 10 metres.

(ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep.

(4)

35. A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

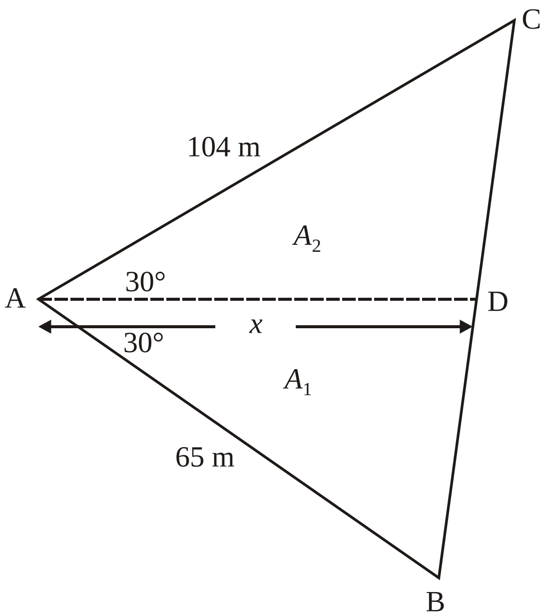
(a) Use the cosine rule to calculate the length of the third side of the field.

(3)

(b) Given that $\sin 60° = \frac{\sqrt{3}}{2}$, find the area of the field in the form $p\sqrt{3}$ where p is an integer.

(3)

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x metres, as shown on the diagram below.



(c) (i) Show that the area of A_1 is given by $\frac{65x}{4}$.

(ii) Find a similar expression for the area of A_2.

(iii) **Hence**, find the value of x in the form $q\sqrt{3}$, where q is an integer.

(7)

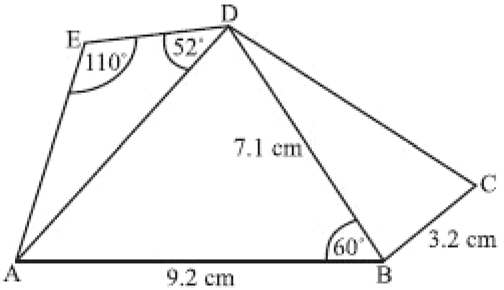
(d) (i) Explain why $\sin A\hat{D}C = \sin A\hat{D}B$.

(ii) Use the result of part (i) and the sine rule to show that

$$\frac{BD}{DC} = \frac{5}{8}.$$

(5)
(Total 18 marks)

36. The following diagram shows a pentagon ABCDE, with AB = 9.2 cm, BC = 3.2 cm, BD = 7.1 cm, $A\hat{E}D$ $=110°$, $A\hat{D}E = 52°$ and $A\hat{B}D = 60°$.



(a) Find AD.

(4)

(b) Find DE.

(4)

(c) The area of triangle BCD is 5.68 cm^2. Find $D\hat{B}C$.

(4)

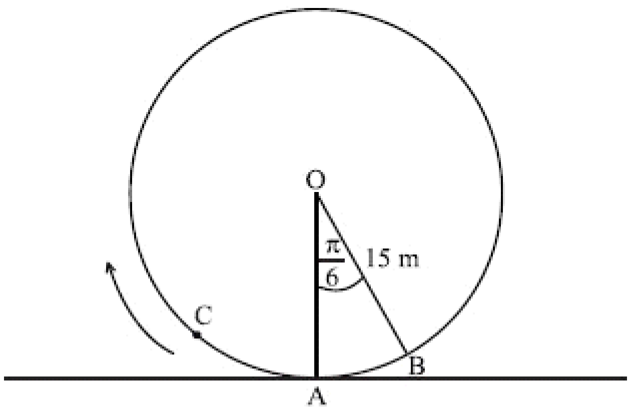
(d) Find AC.

(4)

(e) Find the area of quadrilateral ABCD.

(5)
(Total 21 marks)

37. A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $\hat{AOB} = \frac{\pi}{6}$.



(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector AOB.

(2)

(c) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$. Find the height of A above the ground.

(3)

The height, h metres, of seat C above the ground after t minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right).$$

(d) (i) Find the height of seat C when $t = \frac{\pi}{4}$.

(ii) Find the initial height of seat C.

(iii) Find the time at which seat C first reaches its highest point.

(8)

(e) For $0 \le t \le \pi$,

(i) sketch the graph of h;

(Total 17 marks)