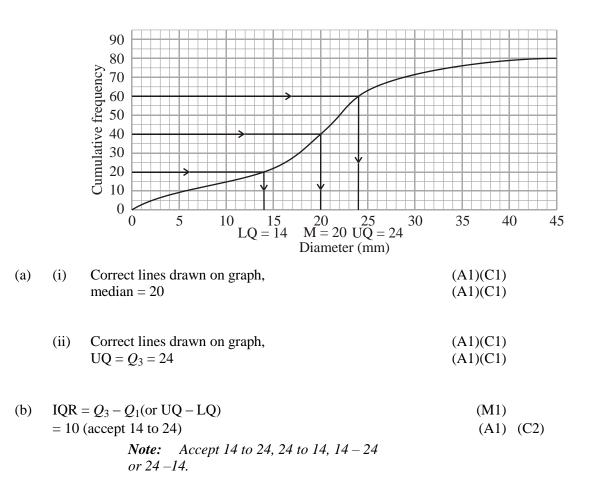
FINAL EXAM RVIEW 2012 - 2013

ANSWER KEY.

1. Mean = 
$$\frac{(72 \times 1.79) + (28 \times 1.62)}{100}$$
 (M1)(M1)  
= 1.7424 (= 1.74 to 3 sf) (A1) (C4)

2. 
$$\frac{(10 \times 1) + (20 \times 2) + (30 \times 5) + (40 \times k) + (50 \times 3)}{k + 11} = 34$$
 (M1)(A1)  
$$\frac{40k + 350}{k + 11} = 34$$
 (A1)  
$$\Rightarrow k = 4$$
 (A1) (C4)

3.



A1 N1

[6]

[4]

(b)	6	A2	N2
(c)	Recognizing the link between 6 and the upper quartile $eg$ 25% scored greater than 6,	(M1)	
	$0.25 \times 32$	(A1)	
	8	A1	N3

5.	(a)	(i) $m = 165$	A1	N1
		(ii) Lower quartile $(1^{st} \text{ quarter}) = 160$ Upper quartile $(3^{rd} \text{ quarter}) = 170$ IQR = 10	(A1) (A1) A1	N3
	(b)	Recognize the need to use the 40 <sup>th</sup> percentile, or 48 <sup>th</sup> student <i>eg</i> a horizontal line through (0, 48) a = 163	(M1) A1	N2

6. (a) (i) 10 (A1)  
(ii) 
$$14 + 10 = 24$$
 (A1)

(ii) 
$$14 + 10 = 24$$
 (A1)

(b)

	<i>x</i> <sub>i</sub>	$f_{\rm i}$	
ſ	15	1	)
	25	5	
	35	7	
	45	9	
(A1)	55	10	(A1)
	65	16	
	75	14	
	85	10	
l	95	8	J
		80	(AG)

Note: Award (A0) for using the mid-interval values of 14.5, 24.5 etc.

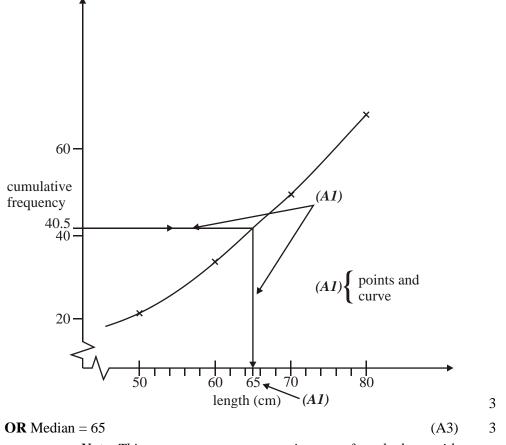
(i)	$\mu = 63$	(A1)

(ii) 
$$\sigma = 20.5 (3 \text{ sf})$$
 (A1) 4

Assymetric diagram/distribution (A1) (c) 1

(d)

[6]



*Note: This answer assumes appropriate use of a calculator with correct arguments.* 

**OR** Linear interpolation on the table: (M1)

$$\left(\frac{48-40.5}{48-32}\right) \times 60 + \left(\frac{40.5-32}{48-32}\right) \times 70 = 65 \text{ (2sf)}$$
 (A1)(A1) 3

[1	0]

**7.** (a)

	Boy	Girl	Total
TV	13	25	38
Sport	33	29	62
Total	46	54	100

$P(TV) = \frac{38}{100}$	(A1) (C2)
--------------------------	-----------

(b) 
$$P(TV | Boy) = \frac{13}{46} (= 0.283 \text{ to } 3 \text{ sf})$$
 (A2) (C2)

*Notes:* Award (A1) for numerator and (A1) for denominator. Accept equivalent answers.

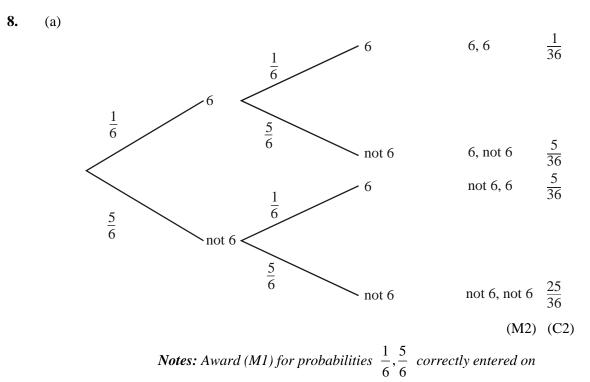


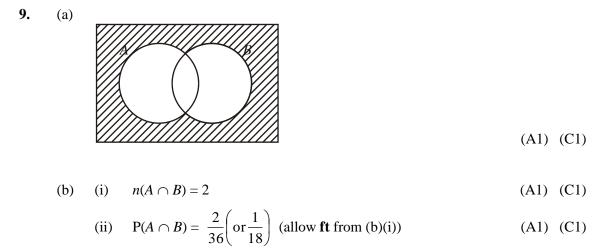
diagram.

Award (M1) for correctly listing the outcomes 6, 6; 6 not 6; not 6, 6; not 6, not 6, or the corresponding probabilities.

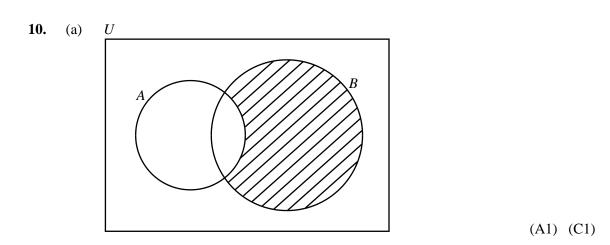
(b) P(one or more sixes) = 
$$\frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6}$$
 or  $\left(1 - \frac{5}{6} \times \frac{5}{6}\right)$  (M1)  
=  $\frac{11}{36}$  (A1) (C2)

[4]

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(c) 
$$n(A \cap B) \neq 0$$
 (or equivalent) (R1) (C1)



(b)  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   $65 = 30 + 50 - n(A \cap B)$   $\Rightarrow n(A \cap B) = 15 \text{ (may be on the diagram)}$   $n(B \cap A') = 50 - 15 = 35$ (M1) (A1) (C2)

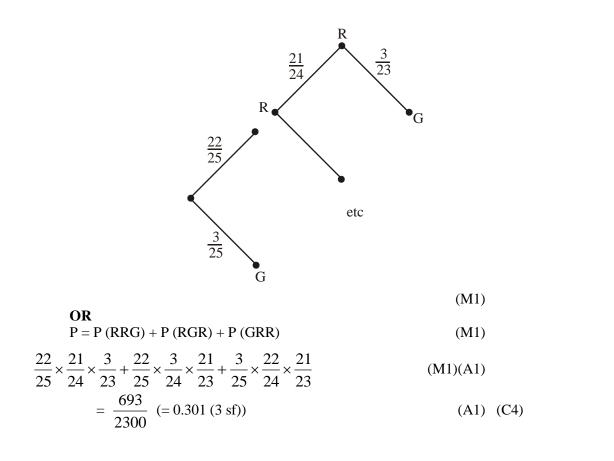
(c) 
$$P(B \cap A') = \frac{n(B \cap A')}{n(U)} = \frac{35}{100} = 0.35$$
 (A1) (C1)

**11.** (a) 
$$P = \frac{22}{23} (= 0.957 (3 \text{ sf}))$$
 (A2) (C2)

(b)

5

[4]



13.	(a)	For taking three ratios of consecutive terms	(M1)	
		$\frac{54}{18} = \frac{162}{54} = \frac{486}{162}  (=3)$	A1	
		hence geometric	AG	N0

[6]

	(b)	(i) (ii)	r = 3 $u_n = 18 \times 3^{n-1}$ For a valid attempt to solve $18 \times 3^{n-1} = 1062882$ <i>eg</i> trial and error, logs n = 11	(A1) A1 (M1) A1	N2 N2	
					1 (2	[6]
14.	eg (	$\binom{10}{8} \times 2$		M1		
	$\begin{pmatrix} 10\\ 8 \end{pmatrix}$	) =	45	(A1)		
		2 × 2,		(A2) A2	N4	[6]
15.	(a)	7 ter	ms	A1	N1	

(b) A valid approach (M1)

Correct term **chosen**  $\binom{6}{3}(x^3)^3(-3x)^3$  A1

Calculating 
$$\binom{6}{3} = 20, (-3)^3 = -27$$
 (A1)(A1)

Term is 
$$-540x^{12}$$
 A1

**16.** (a) 
$$\log_a 10 = \log_a (5 \times 2)$$
 (M1)  
=  $\log_a 5 + \log_a 2$   
=  $p + q$  A1 N2

[6]

N3

(b) 
$$\log_a 8 = \log_a 2^3$$
 (M1)  
=  $3 \log_a 2$ 

$$= 3q$$
 A1 N2

(c) 
$$\log_a 2.5 = \log_a \frac{5}{2}$$
 (M1)  
=  $\log_a 5 - \log_a 2$ 

$$= p - q$$
 A1 N2

[6]

[6]

**17.** (a) 
$$\frac{1}{5}$$
 (0.2) A1 N1

(b) (i) 
$$u_{10} = 25 \left(\frac{1}{5}\right)^9$$
 (M1)

$$= 0.0000128 \left( \left(\frac{1}{5}\right)^7, 1.28 \times 10^{-5}, \frac{1}{78125} \right)$$
A1 N2

(ii) 
$$u_n = 25 \left(\frac{1}{5}\right)^{n-1}$$
 A1 N1

(c) For attempting to use infinite sum formula for a GP  $\left(\frac{25}{1-\left(\frac{1}{5}\right)}\right)$  (M1)

$$S = \frac{125}{4} = 31.25 \ (=31.3 \ \text{to} \ 3s \ f)$$
 A1 N2

**18.** (a) For taking an appropriate ratio of consecutive terms (M1)  

$$r = \frac{2}{3}$$
 A1 N2

	(b)	For attempting to use the formula for the $n^{\text{th}}$ term of a GP $u_{15} = 1.39$	(M1) A1	N2
	(c)	For attempting to use infinite sum formula for a GP $S = 1215$	(M1) A1	N2
19.	(a)	(i) $\log_c 15 = \log_c 3 + \log_c 5$	(A1)	
		= p + q	A1	N2
		(ii) $\log_c 25 = 2 \log_c 5$	(A1)	
		=2q	A1	N2
	(b)	METHOD 1		
		$d^{\frac{1}{2}} = 6$	M1	
		<i>d</i> = 36	A1	N1
		METHOD 2		
		For changing base	M1	
		$eg \qquad \frac{\log_{10} 6}{\log_{10} d} = \frac{1}{2}, 2\log_{10} 6 = \log_{10} d$		
		<i>d</i> = 36	A1	N1
20.	(a)	(i) $r = -2$	A1	N1

(ii) 
$$u_{15} = -3 (-2)^{14}$$
 (A1)  
= -49152 (accept -49200) A1 N2

(b)	(i)	2, 6, 18	A1	N1
	(ii)	<i>r</i> = 3	A1	N1

[6]

[6]

## (c) Setting up equation (or a sketch)

$$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$$
 (or correct sketch with relevant information) A1

M1

$$x^{2} + 2x + 1 = 2x^{2} + 2x - 24$$
(A1)
$$x^{2} = 25$$

$$x = 5 \quad \text{or} \quad x = -5$$

$$x = -5 \quad \text{A1} \quad \text{N2}$$

Notes: If "trial and error" is used, work must be documented with several trials shown. Award full marks for a correct answer with this approach. If the work is **not** documented, award N2 for a correct answer.

(d) (i) 
$$r = \frac{1}{2}$$
 A1 N1

(ii) For attempting to use infinite sum formula for a GP (M1)  

$$S = \frac{-8}{1 - \frac{1}{2}}$$

$$S = -16$$
*Note:* Award M0A0 if candidates use a value of r  
where  $r > 1$ , or  $r < -1$ .

[12]

21. (a) 
$$p = -\frac{1}{2}, q = 2$$
 (A1)(A1) (C2)  
or vice versa

(b) By symmetry *C* is midway between *p*, *q* (M1) *Note: This (M1) may be gained by implication.* 

$$\Rightarrow x \text{-coordinate is } \frac{-\frac{1}{2}+2}{2} = \frac{3}{4}$$
 (A1) (C2)

*Note:* Accept 120°, 240°.

**23.** (a)  $f^{-1}(2) \Rightarrow 3x + 5 = 2$  (M1) x = -1 (A1) (C2)

(b) 
$$g(f(-4) = g(-12 + 5))$$
  
=  $g(-7)$  (A1)  
=  $2(1 + 7)$   
=  $16$  (A1) (C2)

24.	$4x^2 + 4kx + 9 = 0$	
	Only one solution $\Rightarrow b^2 - 4ac = 0$	(M1)
	$16k^2 - 4(4)(9) = 0$	(A1)
	$k^2 = 9$	
	$k = \pm 3$	(A1)
	But given $k > 0$ , $k = 3$	(A1) (C4)
	OR	

One solution  $\Rightarrow (4x^2 + 4kx + 9)$  is a perfect square(M1) $4x^2 + 4kx + 9 = (2x \pm 3)^2$  by inspection(A2)given k > 0, k = 3(A1)

[4]

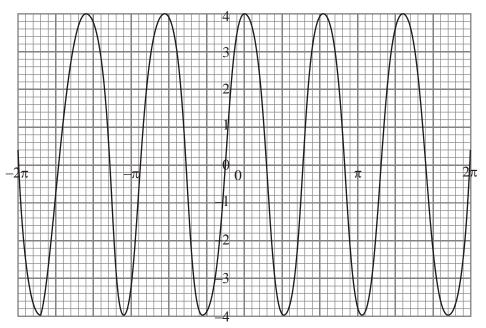
[4]

**25.** From sketch of graph  $y = 4 \sin\left(3x + \frac{\pi}{2}\right)$ 

or by observing 
$$|\sin \theta| \le 1$$
  
 $k > 4, k < -4$ 



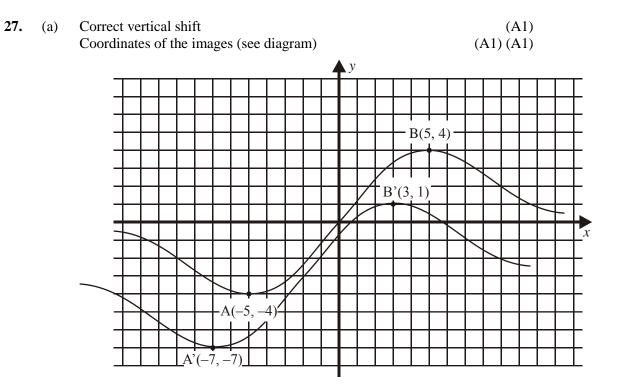
(A1)(A1)(C2)(C2)



26.	(a)	$f(x) = x^2 - 6x + 14$	
		$f(x) = x^2 - 6x + 9 - 9 + 14$	(M1)
		$f(x) = (x-3)^2 + 5$	(M1)

(b) Vertex is (3, 5)

(A1)(A1)

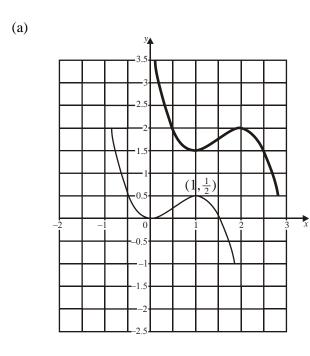


(b) Asymptote: y = -3

28.

(A1)

[4]



(A2) (C2)

13

(b) Minimum: 
$$(1, \frac{3}{2})$$
 (A1) (C1)  
Maximum: (2, 2) (A1) (C1)

**29.** (a) 
$$f(3) = 2^3$$
 (M1)

$$(g \circ f)(3) = \frac{2^{5}}{2^{3} - 2}$$
(M1)

$$=\frac{8}{6}$$
(A1)

$$(g \circ f)(3) = \frac{4}{3}$$
 (C3)

(b) 
$$x = \frac{y}{y-2}$$
 (M1)  
 $x (y-2) = y \Rightarrow y (x-1) = 2x$   
 $\Rightarrow y = \frac{2x}{(x-1)}$  (A1)  
 $y = \frac{10}{(5-1)} = 2.5$  (A1) (C3)

**30.** (a) 
$$2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8$$
 (M1)  
=  $2(x - 2)^2 - 3$  (A1)(A1)(A1)  
=>  $a = 2, p = 2, q = -3$  (C4)

(b)	Minimum value of $2(x-2)^2 = 0$ (or minimum value occurs w	when $x = 2$ (Ml)
	$\Rightarrow$ Minimum value of $f(x) = -3$	(A1) (C2)
	OR	
	Minimum value occurs at $(2, -3)$	(M1)(A1) (C2)

[6]

14

[4]

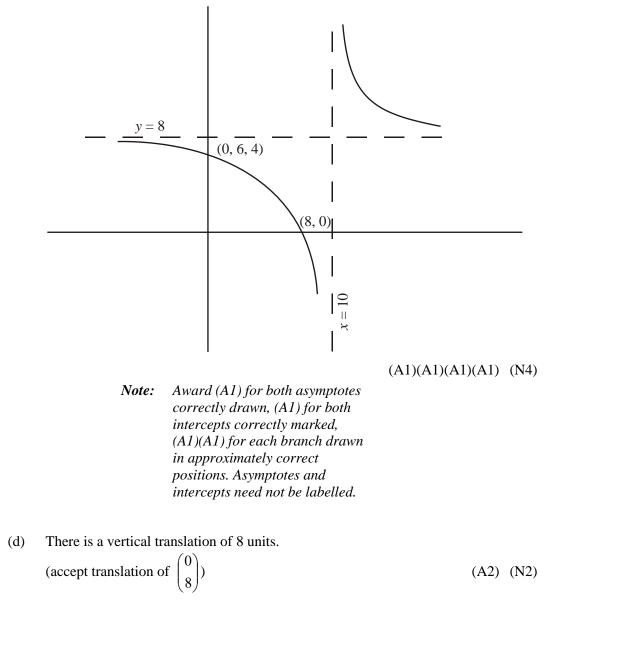
[6]

31.	Discriminant $\Delta = b^2 - 4ac \ (= (-2k)^2 - 4)$ $\Delta > 0$ <i>Note:</i> Award (M1)(M0) for $\Delta \ge 0$ .	(A1) (M2)
	$(2k)^2 - 4 > 0 \Longrightarrow 4k^2 - 4 > 0$	
	EITHER	
	$4k^2 > 4 \ (k^2 > 1)$	(A1)
	OR	
	4(k-1)(k+1) > 0	(A1)
	OR	
	(2k-2)(2k+2) > 0	(A1)

## THEN

k < -1 or $k > 1$		(A1)(A1) (C6)
	<i>Note:</i> Award (A1) for $-1 < k < 1$ .	

(i)	x = 10	(A1) (N1)
(ii)	<i>y</i> = 8	(A1) (N1)
(i)	6.4 (or (0, 6.4))	(A1) (N1)
(ii)	8 (or (8, 0))	(A1) (N1)
	(ii) (i)	(i) $x = 10$ (ii) $y = 8$ (i) $6.4$ (or $(0, 6.4)$ ) (ii) $8$ (or $(8, 0)$ )



**33.** (a) f(1) = 3

(c)

f(5) = 3

(A1)(A1) 2

[10]

(b)	EITHER	distance between successive maxima = period	(M1)
		= 5 - 1	(A1)
		=4	(AG)

**OR** Period of 
$$\sin kx = \frac{2\pi}{k}$$
; (M1)

so period = 
$$\frac{2\pi}{\frac{\pi}{2}}$$
 (A1)

$$= 4 \tag{AG} 2$$

(c) **EITHER** 
$$A \sin\left(\frac{\pi}{2}\right) + B = 3$$
 and  $A \sin\left(\frac{3\pi}{2}\right) + B = -1$  (M1) (M1)  
 $\Rightarrow A + B = 3, -A + B = -1$  (A1)(A1)  
 $\Rightarrow A = 2, B = 1$  (AG)(A1)  
**OR** Amplitude  $= A$  (M1)  
 $A = \frac{3 - (-1)}{2} = \frac{4}{2}$  (M1)  
 $A = 2$  (AG)  
Midpoint value  $= B$  (M1)  
 $B = \frac{3 + (-1)}{2} = \frac{2}{2}$  (M1)  
 $B = 1$  (A1)

*Note:* As the values of A = 2 and B = 1 are likely to be quite obvious to a bright student, do not insist on too detailed a proof.

(d) 
$$f(x) = 2\sin\left(\frac{\pi}{2}x\right) + 1$$
  

$$f'(x) = \left(\frac{\pi}{2}\right)2\cos\left(\frac{\pi}{2}x\right) + 0$$
 (M1)(A2)  
Note: Award (M1) for the chain rule, (A1) for  $\left(\frac{\pi}{2}\right)$ , (A1) for  

$$2\cos\left(\frac{\pi}{2}x\right).$$
  

$$= \pi\cos\left(\frac{\pi}{2}x\right)$$
 (A1)

**Notes:** Since the result is given, make sure that reasoning is valid. In particular, the final (A1) is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of "fudged" results.

(e) (i) 
$$y = k - \pi x$$
 is a tangent  $\Rightarrow -\pi = \pi \cos\left(\frac{\pi}{2}x\right)$  (M1)  
 $\Rightarrow -1 = \cos\left(\frac{\pi}{2}x\right)$  (A1)

$$\Rightarrow \frac{\pi}{2} x = \pi \text{ or } 3\pi \text{ or } \dots$$
  
$$\Rightarrow x = 2 \text{ or } 6 \dots$$
(A1)

5

4

Since $0 \le x \le 5$ , we take $x = 2$ , so the point is $(2, 1)$	(A1)
--	------

(ii) Tangent line is: 
$$y = -\pi(x - 2) + 1$$
 (M1)  
 $y = (2\pi + 1) - \pi x$   
 $k = 2\pi + 1$  (A1) 6

(f) 
$$f(x) = 2 \Longrightarrow 2 \sin\left(\frac{\pi}{2}x\right) + 1 = 2$$
 (A1)

$$\Rightarrow \sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$$

$$\Rightarrow \frac{\pi}{2}x = \frac{\pi}{6} \operatorname{or} \frac{5\pi}{6} \operatorname{or} \frac{13\pi}{6}$$
(A1)

$$x = \frac{1}{3} \operatorname{or} \frac{5}{3} \operatorname{or} \frac{13}{3}$$
(A1)(A1)(A1) 5

[24]

**34.** (a) (i) 
$$Q = \frac{1}{2}(14.6 - 8.2)$$
 (M1)  
= 3.2 (A1)

(ii) 
$$P = \frac{1}{2}(14.6 + 8.2)$$
 (M0)  
= 11.4 (A1) 3

(b) 
$$10 = 11.4 + 3.2 \cos\left(\frac{\pi}{6}t\right)$$
 (M1)  
so  $\frac{-7}{16} = \cos\left(\frac{\pi}{6}t\right)$ 

therefore 
$$\operatorname{arccos}\left(\frac{-7}{16}\right) = \frac{\pi}{6}t$$
 (A1)

which gives 2.0236... =  $\frac{\pi}{6} t$  or t = 3.8648. t = 3.86(3 sf) (A1) 3

(c)	(i)	By symmetry, next time is $12 - 3.86 = 8.135 t = 8.14$ (3 sf)	(A1)
	(ii)	From above, first interval is $3.86 < t < 8.14$	(A1)
		This will happen again, 12 hours later, so $15.9 < t < 20.1$	(M1) (A1)

**35.** (a) using the cosine rule  $(A2) = b^2 + c^2 - 2bc \cos \hat{A}$  (M1) substituting correctly  $BC^2 = 65^2 + 104^2 - 2(65)(104) \cos 60^\circ$  A1 = 4225 + 10816 - 6760 = 8281 $\Rightarrow BC = 91 \text{ m}$  A1

(b) finding the area, using 
$$\frac{1}{2}bc\sin{\hat{A}}$$
 (M1)

substituting correctly, area = 
$$\frac{1}{2}(65)(104) \sin 60^{\circ}$$
 A1

 = 1690  $\sqrt{3}$  (Accept  $p = 1690$ )
 A1

(c) (i) 
$$A_1 = \left(\frac{1}{2}\right)(65) (x) \sin 30^\circ$$
 A1  
=  $\frac{65x}{4}$  AG 1

(ii) 
$$A_2 = \left(\frac{1}{2}\right)(104) (x) \sin 30^\circ$$
 M1  
= 26x A1 2

(iii) starting  $A_1 + A_2 = A$  or substituting  $\frac{65x}{4} + 26x = 1690\sqrt{3}$  (M1) simplifying  $\frac{169x}{4} = 1690\sqrt{3}$  A1

$$x = \frac{4 \times 1690\sqrt{3}}{169}$$
 A1

$$\Rightarrow x = 40\sqrt{3} \quad (\text{Accept } q = 40) \qquad \qquad \text{A1} \qquad 4$$

(d) (i) Recognizing that supplementary angles have equal sines  
eg 
$$\hat{ADC} = 180 - \hat{ADB} \implies \sin \hat{ADC} = \sin \hat{ADB}$$
 R1

[10]

4

3

## (ii) using sin rule in $\triangle$ ADB and $\triangle$ ACD

substituting correctly  $\frac{BD}{\sin 30^\circ} = \frac{65}{\sin A\hat{D}B} \Rightarrow \frac{BD}{65} = \frac{\sin 30^\circ}{\sin A\hat{D}B}$  A1

(M1)

5

A1

(M1)

(M1)

and 
$$\frac{DC}{\sin 30^{\circ}} = \frac{104}{\sin A\hat{D}B} \Rightarrow \frac{DC}{104} = \frac{\sin 30^{\circ}}{\sin A\hat{D}C}$$
 M1

since sin  $\hat{ADB} = \sin \hat{ADC}$ 

$$\frac{BD}{65} = \frac{DC}{104} \Longrightarrow \frac{BD}{DC} = \frac{65}{104}$$
A1

$$\Rightarrow \frac{BD}{DC} = \frac{5}{8}$$
 AG

[18]

36.	(a)	Evidence of choosing cosine rule	(M1)	
		$eg a^2 = b^2 + c^2 - 2bc \cos A$		
		Correct substitution	A1	
		$eg (AD)^2 = 7.1^2 + 9.2^2 - 2(7.1) (9.2) \cos 60^\circ$		
		$(AD)^2 = 69.73$	(A1)	
		AD = 8.35 (cm)	A1	N2

(b) 
$$180^{\circ} - 162^{\circ} = 18^{\circ}$$
 (A1)  
Evidence of choosing sine rule (M1)

Evidence of choosing sine rule Correct substitution

$$eg \frac{DE}{\sin 18^{\circ}} = \frac{8.35}{\sin 110^{\circ}}$$
  
DE = 2.75 (cm) A1 N2

(c) Setting up equation

 $eg \ \frac{1}{2} \ ab \sin C = 5.68, \ \frac{1}{2} \ bh = 5.68$ Correct substitution A1  $eg \ 5.68 = \ \frac{1}{2} (3.2) (7.1) \sin \ DBC, \ \frac{1}{2} \times 3.2 \times h = 5.68, (h = 3.55)$ 

$$\sin \hat{DBC} = 0.5 \tag{A1}$$

(d) Finding  $A \hat{B} C (60^\circ + D \hat{B} C)$  (A1)

Using appropriate formula

$$eg (AC)^{2} = (AB)^{2} + (BC)^{2}, (AC)^{2} = (AB)^{2} + (BC)^{2} - 2 (AB)$$

(BC) cos ABC

Correct substitution (allow $FT$ on their seen $\hat{ABC}$ )
$eg (AC)^2 = 9.2^2 + 3.2^2$
AC = 9.74 (cm)

(e)	For finding area of triangle ABD	(M1)	
	Correct substitution Area = $\frac{1}{2} \times 9.2 \times 7.1 \sin 60^{\circ}$	A1	
	= 28.28	A1	
	Area of ABCD = 28.28 + 5.68	(M1)	
	$= 34.0 \ (\text{cm}^2)$	A1	N3

A1 A1

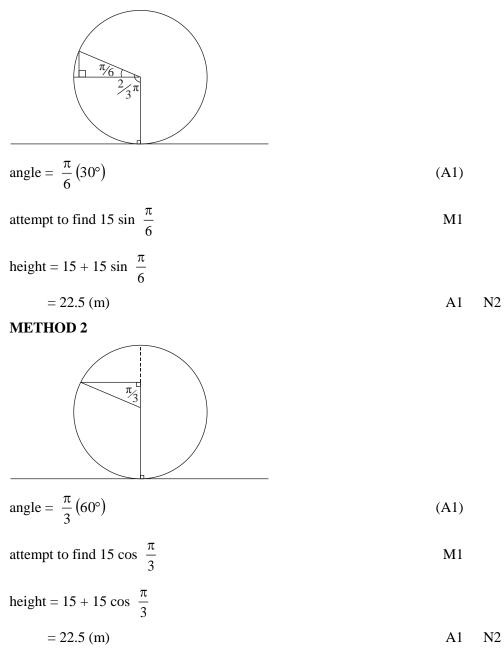
N3

[21]

37.		<i>Note:</i> Accept <i>exact</i> answers given in terms of $\pi$ .		
	(a)	Evidence of using $l = \mathbf{r}\theta$	(M1)	
		arc $AB = 7.85$ (m)	A1	N2

(b) Evidence of using 
$$A = \frac{1}{2}r^2\theta$$
 (M1)

Area of sector AOB =  $58.9 \text{ (m}^2$ ) A1 N2



(d) (i) 
$$h\left(\frac{\pi}{4}\right) = 15 - 15\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$$
 (M1)  
= 25.6 (m) A1 N2

(ii) 
$$h(0) = 15 - 15 \cos\left(0 + \frac{\pi}{4}\right)$$
 (M1)  
= 4.39(m) A1 N2

$$= 4.39(m)$$
 A1

## (iii) METHOD 1

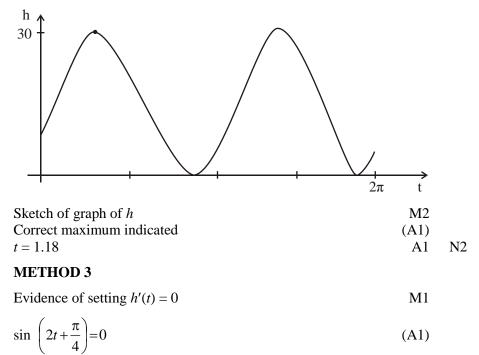
Highest point when h = 30

$$30 = 15 - 15 \cos\left(2t + \frac{\pi}{4}\right) \tag{M1}$$

$$\cos\left(2t + \frac{\pi}{4}\right) = -1\tag{A1}$$

$$t = 1.18 \left( \operatorname{accept} \frac{3\pi}{8} \right)$$
 A1 N2

**METHOD 2** 



Justification of maximum

eg reasoning from diagram, first derivative test, second derivative test

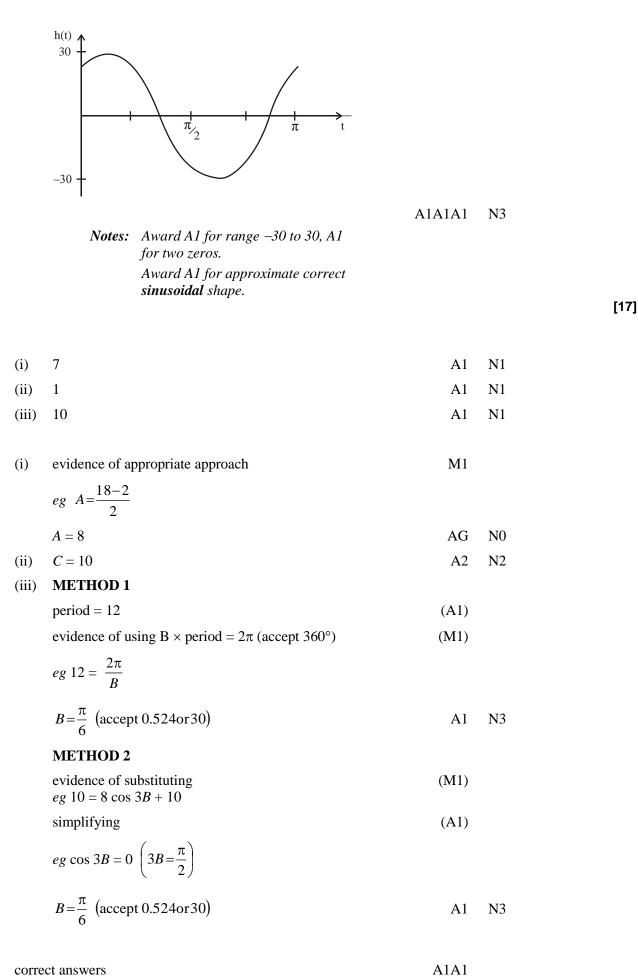
$$t = 1.18 \left( \operatorname{accept} \frac{3\pi}{8} \right)$$
 A1 N2

(i)

R1

R1

(e)



(c) correct answers

38.

(a)

(b)

*eg t* = 3.52, *t* = 10.5, between 03:31 and 10:29 (accept 10:30)

N2