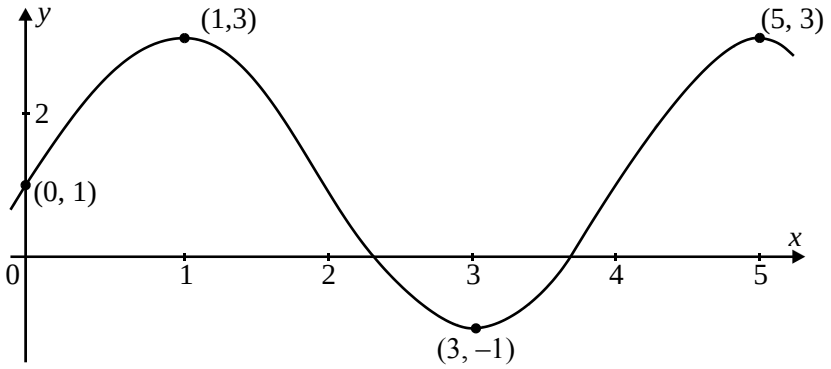
IB Standard Level Mathematics

Trigonometry Modeling (Question Bank)

1. The diagram shows the graph of the function f given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \le x \le 5$, where A and B are constants, and x is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a) Write down the values of $f(1)$ and $f(5)$.

(2)

(b) Show that the period of f is 4.

(2)

The point (3, –1) is a minimum point of the graph.

(c) Show that $A = 2$, and find the value of B.

(5)

(d) Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(4)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \le x \le 5$.

(e) Find

(i) the point where this tangent meets the curve;

(ii) the value of k.

(6)

(f) Solve the equation $f(x) = 2$ for $0 \le x \le 5$.

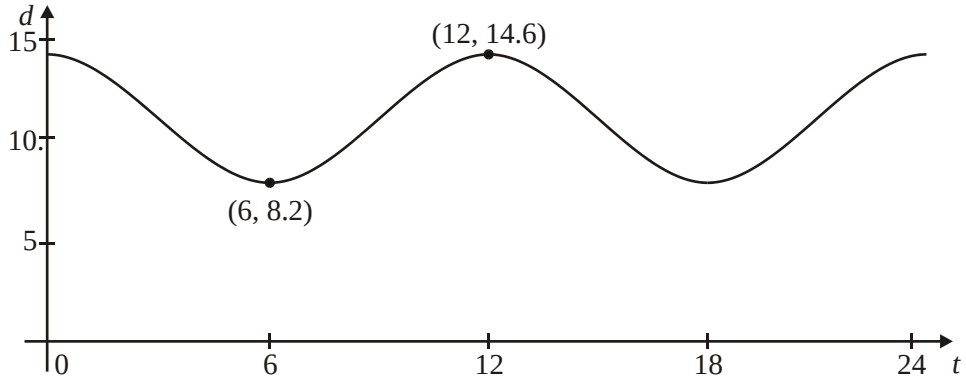
(5)

(Total 24 marks)

2. A formula for the depth d metres of water in a harbour at a time t hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where P and Q are positive constants. In the following graph the point (6, 8.2) is a minimum point and the point (12, 14.6) is a maximum point.



(a) Find the value of

(i) Q;

(ii) P.

(3)

(b) Find the **first** time in the 24-hour period when the depth of the water is 10 metres.

(3)

(c) (i) Use the symmetry of the graph to find the **next** time when the depth of the water is 10 metres.

(ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep.

(4)

3. The depth, y metres, of sea water in a bay t hours after midnight may be represented by the function

$$y = a + b\cos\left(\frac{2\pi}{k}t\right), \text{ where } a, b \text{ and } k \text{ are constants.}$$

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of

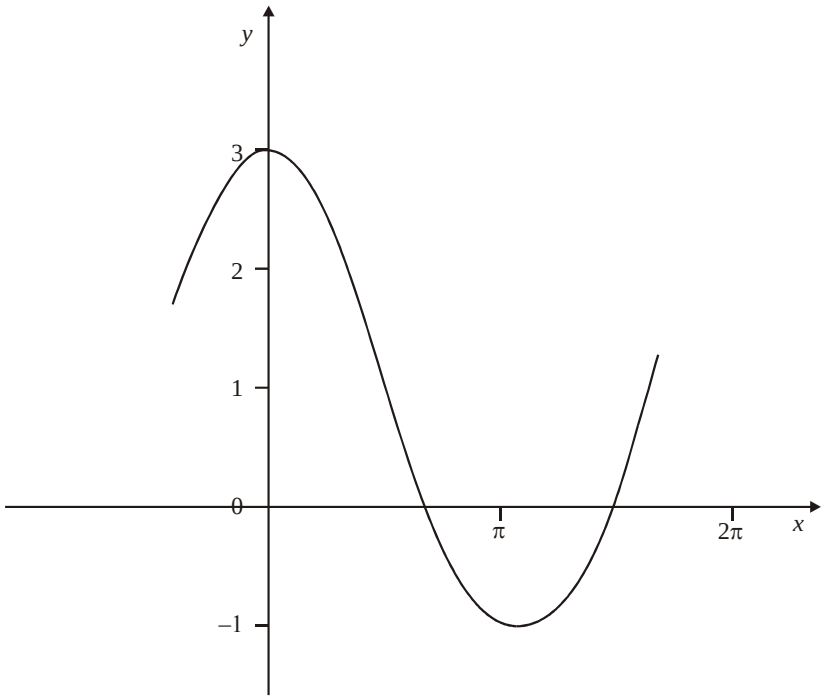
(a) a;

(b) b;

(c) k.

(Total 4 marks)

4. Part of the graph of $y = p + q \cos x$ is shown below. The graph passes through the points $(0, 3)$ and $(\pi, -1)$.



Find the value of

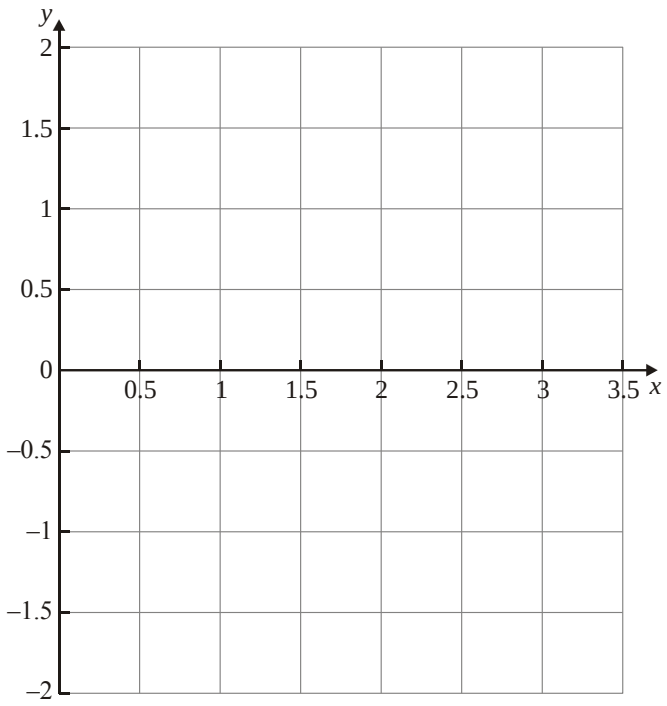
(a) p;

(b) q.

(Total 6 marks)

5. Let $f(x) = \sin(2x + 1)$, $0 \leq x \leq \pi$.

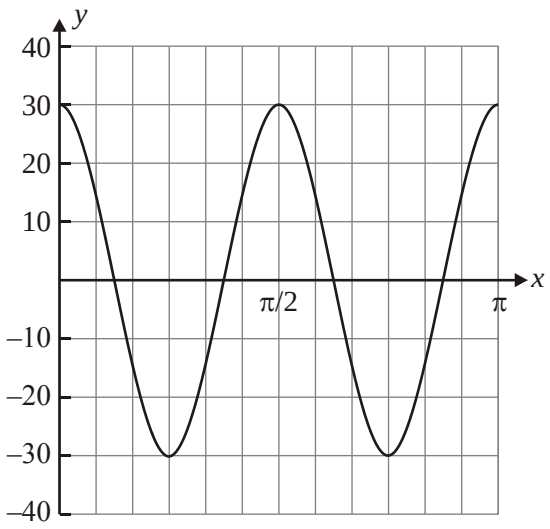
(a) Sketch the curve of $y = f(x)$ on the grid below.



(b) Find the x-coordinates of the maximum and minimum points of $f(x)$, giving your answers correct to one decimal place.

(Total 6 marks)

6. The graph of a function of the form $y = p \cos qx$ is given in the diagram below.

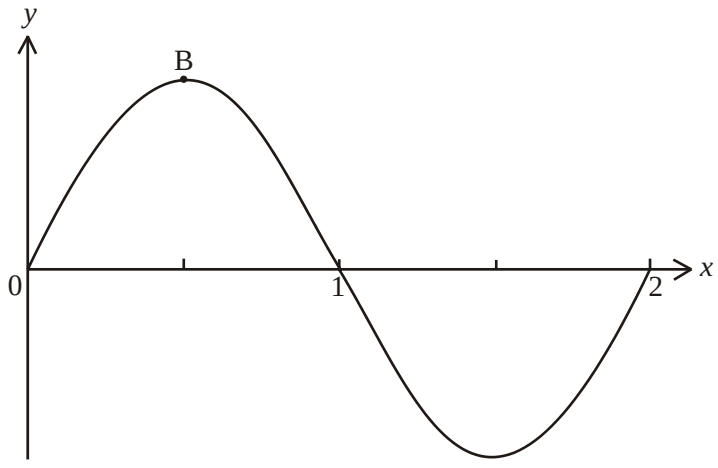


(a) Write down the value of p.

(b) Calculate the value of q.

(Total 6 marks)

7. Let $f(x) = 6 \sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of f is shown on the diagram below. There is a maximum value at B (0.5, b).



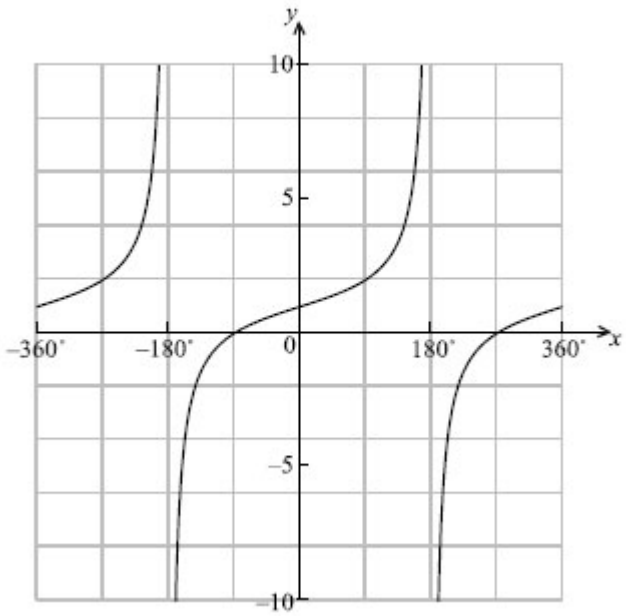
(a) Write down the value of b.

(b) On the same diagram, sketch the graph of g.

(c) Solve $f(x) = g(x)$, $0.5 \le x \le 1.5$.

(Total 6 marks)

8. The diagram below shows the graph of $f(x) = 1 + \tan\left(\frac{x}{2}\right)$ for $-360° \le x \le 360°$.



(a) On the same diagram, draw the asymptotes.

(2)

(b) Write down

(i) the period of the function;

(ii) the value of $f(90°)$.

(2)

(c) Solve $f(x) = 0$ for $-360° \le x \le 360°$.

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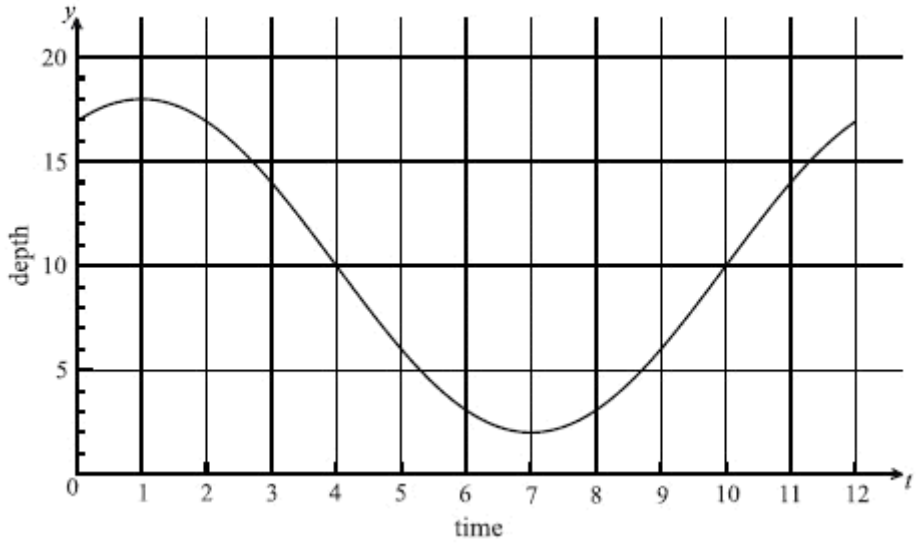
(2)

(Total 6 marks)

9. The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

(3)

(b) The depth of water can be modelled by the function $y = A\cos(B(t-1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B.

(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P.

(2)

(Total 11 marks)