

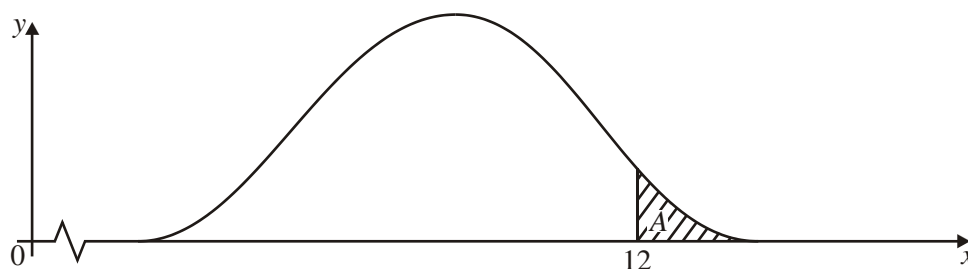
IB MATHEMATICS STANDARD LEVEL

Normal Probability Distribution

NAME: _____

DATE: _____

1. The graph shows a normal curve for the random variable X , with mean μ and standard deviation σ .



It is known that $p(X \geq 12) = 0.1$.

- (a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A .

(1)

It is also known that $p(X \leq 8) = 0.1$.

- (b) Find the value of μ , explaining your method in full.

(5)

- (c) Show that $\sigma = 1.56$ to an accuracy of three significant figures.

(5)

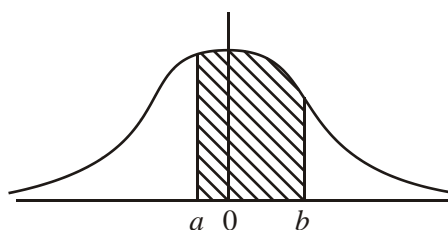
- (d) Find $p(X \leq 11)$.

(5)

(Total 16 marks)

2. The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

- (a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of a and b . (2)

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours; (1)

(b) between 55 and 60 hours. (2)

(b) 90% of the insects die after t hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%. (2)

(ii) Find the value of t . (3)

(Total 10 marks)

3. An urban highway has a speed limit of 50 km h^{-1} . It is known that the speeds of vehicles travelling on the highway are normally distributed, with a standard deviation of 10 km h^{-1} , and that 30% of the vehicles using the highway exceed the speed limit.

(a) Show that the mean speed of the vehicles is approximately 44.8 km h^{-1} . (3)

The police conduct a “Safer Driving” campaign intended to encourage slower driving, and want to know whether the campaign has been effective. It is found that a sample of 25 vehicles has a mean speed of 41.3 km h^{-1} .

(b) Given that the null hypothesis is

H_0 : the mean speed has been unaffected by the campaign

State H_1 , the alternative hypothesis.

(1)

(c) State whether a one-tailed or two-tailed test is appropriate for these hypotheses, and explain why. (2)

(d) Has the campaign had significant effect at the 5% level?

(4)
(Total 10 marks)

4. Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.

(a) What percentage of the population has an IQ between 90 and 125?

(2)

(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125?

(3)

(c) The mean IQ of a random group of 25 persons suffering from a certain brain disorder was found to be 95.2. Is this sufficient evidence, at the 0.05 level of significance, that people suffering from the disorder have, on average, a lower IQ than the entire population? State your null hypothesis and your alternative hypothesis, and explain your reasoning.

(4)
(Total 9 marks)

5. Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg?

(2)

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg.

(3)

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.

- (c) Calculate the mean and standard deviation that satisfy these requirements.

(3)

The cost of the new machine is \$5000. Cement sells for \$0.80 per kg.

- (d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment?

(3)

(Total 11 marks)

6. The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

- (a) Find the probability that a packet chosen at random has mass

- (i) less than 740 g;
- (ii) at least 780 g;
- (iii) between 740 g and 780 g.

(5)

- (b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g?

(2)

- (c) The mass of 70% of the packets is more than x grams. Find the value of x .

(2)

(Total 9 marks)

7. In a country called *Tallopia*, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

- (a) What percentage of adults in *Tallopia* have a height greater than 197 cm?

(3)

- (b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm.

(4)

(Total 7 marks)

8. A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

- (a) What proportion of television sets break down in less than 72 months?

(2)

- (b) (i) Calculate the proportion of sets which have a lifetime between 72 months and 90 months.

- (ii) Illustrate this proportion by appropriate shading in a sketch of a normal distribution curve.

(5)

- (c) If a set breaks down in less than x months, the company replace it free of charge. They replace 4% of the sets. Find the value of x .

(3)

(Total 10 marks)