

## STATS AND PROBABILITY REVIEW

### REVIEW QUESTION #1:

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M11/5/MATME/SP2/ENG/TZ1/XX

Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

10. [Maximum mark: 16]

Let  $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$ , for  $x > 0$ .

(a) Show that  $f(x) = \log_3 2x$ . [2 marks]

(b) Find the value of  $f(0.5)$  and of  $f(4.5)$ . [3 marks]

The function  $f$  can also be written in the form  $f(x) = \frac{\ln ax}{\ln b}$ .

(c) (i) Write down the value of  $a$  and of  $b$ .

(ii) Hence on graph paper, **sketch** the graph of  $f$ , for  $-5 \leq x \leq 5$ ,  $-5 \leq y \leq 5$ , using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote. [6 marks]

(d) Write down the value of  $f^{-1}(0)$ . [1 mark]

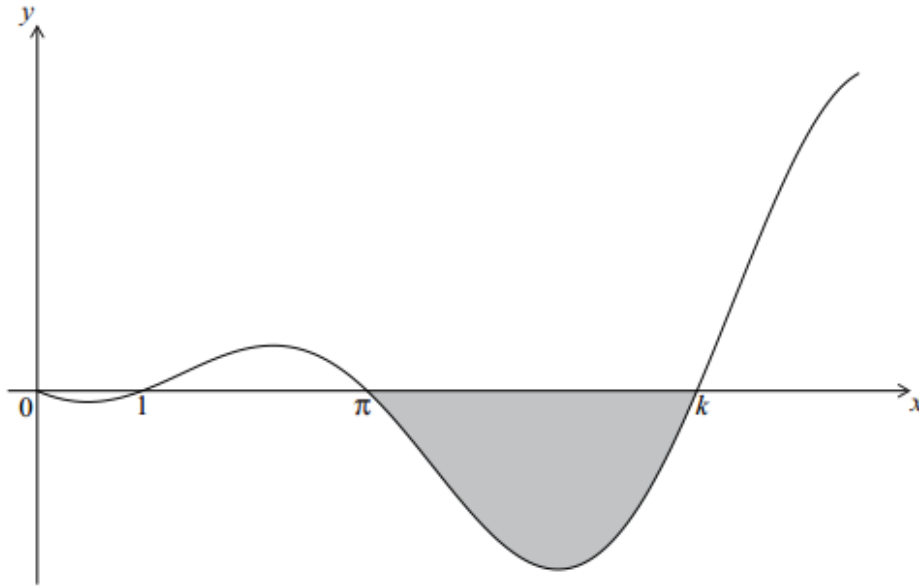
The point A lies on the graph of  $f$ . At A,  $x = 4.5$ .

(e) On your diagram, sketch the graph of  $f^{-1}$ , noting clearly the image of point A. [4 marks]

## REVIEW QUESTION #2

4. [Maximum mark: 7]

The graph of  $y = (x-1)\sin x$ , for  $0 \leq x \leq \frac{5\pi}{2}$ , is shown below.



The graph has  $x$ -intercepts at 0, 1,  $\pi$  and  $k$ .

(a) Find  $k$ .

[2 marks]

The shaded region is rotated  $360^\circ$  about the  $x$ -axis. Let  $V$  be the volume of the solid formed.

(b) Write down an expression for  $V$ .

[3 marks]

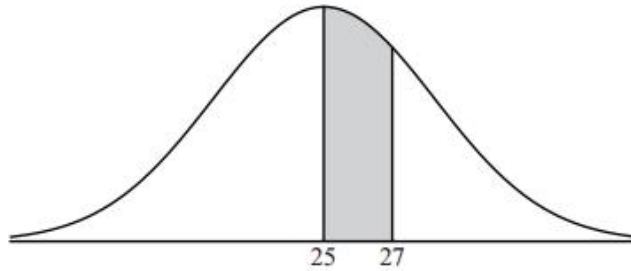
(c) Find  $V$ .

[2 marks]

## STATISTICS AND PROBABILITY REVIEW

6. [Maximum mark: 7]

Let the random variable  $X$  be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find  $P(X > 27)$ .

[2 marks]

(b) Find the standard deviation of  $X$ .

[5 marks]

9. [Maximum mark: 12]

Two fair 4-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) List the pairs of scores that give a sum of 6.

[3 marks]

The probability distribution for the sum of the scores on the two dice is shown below.

| Sum         | 2   | 3   | 4              | 5              | 6              | 7   | 8              |
|-------------|-----|-----|----------------|----------------|----------------|-----|----------------|
| Probability | $p$ | $q$ | $\frac{3}{16}$ | $\frac{4}{16}$ | $\frac{3}{16}$ | $r$ | $\frac{1}{16}$ |

(b) Find the value of  $p$ , of  $q$ , and of  $r$ .

[3 marks]

Fred plays a game. He throws two fair 4-sided dice four times. He wins a prize if the sum is 5 on three or more throws.

(c) Find the probability that Fred wins a prize.

[6 marks]

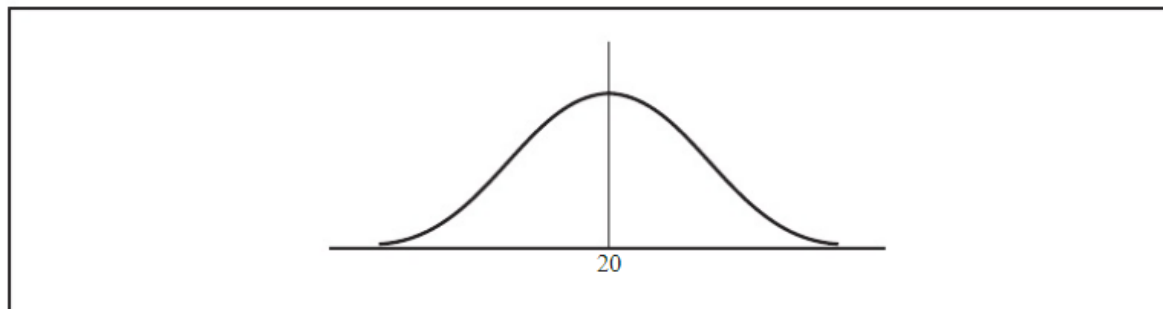
4. [Maximum mark: 8]

A random variable  $X$  is distributed normally with a mean of 20 and variance 9.

(a) Find  $P(X \leq 24.5)$ . [3 marks]

(b) Let  $P(X \leq k) = 0.85$ .

(i) Represent this information on the following diagram.



(ii) Find the value of  $k$ . [5 marks]

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5. [Maximum mark: 7]

A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box. [2 marks]

(b) Find the probability that there are 15 brown eggs in the box. [2 marks]

(c) Find the probability that there are at least 10 brown eggs in the box. [3 marks]

7. [Maximum mark: 7]

A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.

90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection. [3 marks]

(b) The company would like the probability that a box passes inspection to be 0.87. Find the percentage of boxes that should be made by machine B to achieve this. [4 marks]

7. [Maximum mark: 7]

The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining

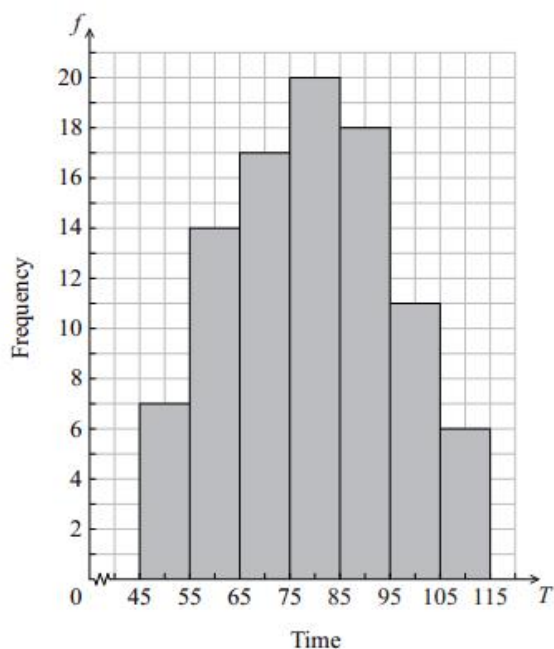
(a) **at least** four tails; [4 marks]

(b) the fourth tail on the tenth toss. [3 marks]

Answer **all** questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 13]

The histogram below shows the time  $T$  seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for  $T$ .

| Time      | $45 \leq T < 55$ | $55 \leq T < 65$ | $65 \leq T < 75$ | $75 \leq T < 85$ | $85 \leq T < 95$ | $95 \leq T < 105$ | $105 \leq T < 115$ |
|-----------|------------------|------------------|------------------|------------------|------------------|-------------------|--------------------|
| Frequency | 7                | 14               | $p$              | 20               | 18               | $q$               | 6                  |

(a) (i) Write down the value of  $p$  and of  $q$ .

- (ii) Write down the median class. [3 marks]
- (b) A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle. [2 marks]

(This question continues on the following page)

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Do **NOT** write solutions on this page.

(Question 8 continued)

Consider the class interval  $45 \leq T < 55$ .

- (c) (i) Write down the interval width.
- (ii) Write down the mid-interval value. [2 marks]
- (d) Hence find an estimate for the
- (i) mean;
- (ii) standard deviation. [4 marks]

John assumes that  $T$  is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.

- (e) Find John's estimate. [2 marks]

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4. [Maximum mark: 6]

The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

- (a) Find the probability that this child is taller than 122.5 cm. [3 marks]
- (b) The probability that this child is shorter than  $k$  cm is 0.65. Find the value of  $k$ . [3 marks]

7. [Maximum mark: 8]

A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

- (a) Find the probability that there is at least one defective lamp in the sample. [4 marks]
- (b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. [4 marks]

1. [Maximum mark: 16]

The heat output in thermal units from burning 1 kg of wood changes according to the wood's percentage moisture content. The moisture content and heat output of 10 blocks of the same type of wood each weighing 1 kg were measured. These are shown in the table.

|  |    |    |    |    |    |    |    |    |    |    |
|--|----|----|----|----|----|----|----|----|----|----|
| <b>Moisture content % (<math>x</math>)</b> | 8  | 15 | 22 | 30 | 34 | 45 | 50 | 60 | 74 | 82 |
| <b>Heat output (<math>y</math>)</b>        | 80 | 77 | 74 | 69 | 68 | 61 | 61 | 55 | 50 | 45 |

- (a) Draw a scatter diagram to show the above data. Use a scale of 2 cm to represent 10 % on the  $x$ -axis and a scale of 2 cm to represent 10 thermal units on the  $y$ -axis. [4 marks]
- (b) Write down
- (i) the mean percentage moisture content,  $\bar{x}$ ;
- (ii) the mean heat output,  $\bar{y}$ . [2 marks]
- (c) Plot the point  $(\bar{x}, \bar{y})$  on your scatter diagram and label this point M. [2 marks]
- (d) Write down the product-moment correlation coefficient,  $r$ . [2 marks]

The equation of the regression line  $y$  on  $x$  is  $y = -0.470x + 83.7$ .

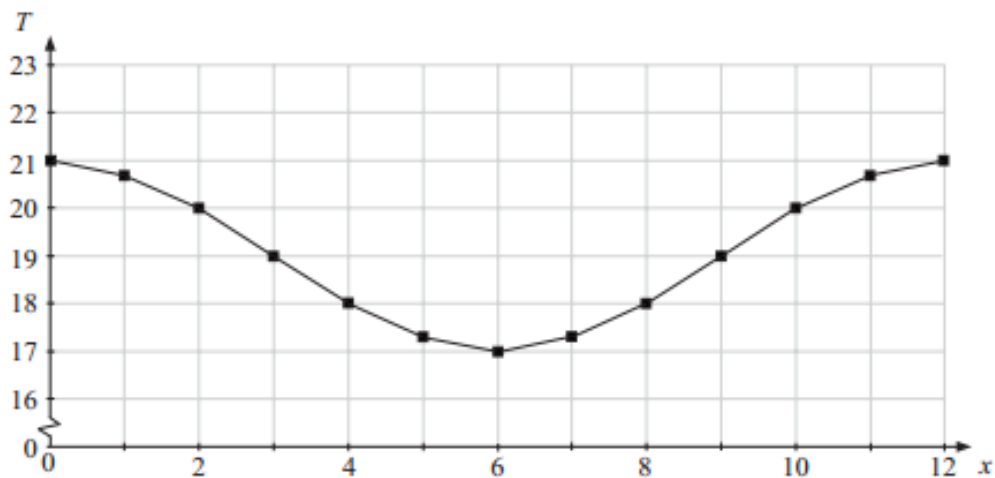
- (e) Draw the regression line  $y$  on  $x$  on your scatter diagram. [2 marks]
- (f) Estimate the heat output in thermal units of a 1 kg block of wood that has 25 % moisture content. [2 marks]
- (g) State, with a reason, whether it is appropriate to use the regression line  $y$  on  $x$  to estimate the heat output in part (f). [2 marks]

3. [Maximum mark: 13]

- (i) The graph below represents the temperature ( $T^\circ$  Celsius) in Washington measured at midday during a period of thirteen consecutive days starting at Day 0. These points also lie on the graph of the function

$$T(x) = a + b \cos(cx^\circ), \quad 0 \leq x \leq 12,$$

where  $a$ ,  $b$  and  $c \in \mathbb{R}$ .



- (a) Find the value of
- $a$ ;
  - $b$ . [2 marks]
- (b) Show that  $c = 30$ . [1 mark]
- (c) Using the graph, or otherwise, write down the part of the domain for which the midday temperature is less than  $18.5^\circ$ . [2 marks]

(This question continues on the following page)



(Question 3 continued)

- (ii) The number of bottles of water sold at a railway station on each day is given in the following table.

| Day                            | 0   | 1    | 2   | 3   | 4  | 5    | 6  | 7    | 8   | 9   | 10  | 11   | 12  |
|--------------------------------|-----|------|-----|-----|----|------|----|------|-----|-----|-----|------|-----|
| Temperature ( $T^{\circ}$ )    | 21  | 20.7 | 20  | 19  | 18 | 17.3 | 17 | 17.3 | 18  | 19  | 20  | 20.7 | 21  |
| Number of bottles sold ( $n$ ) | 150 | 141  | 126 | 125 | 98 | 101  | 93 | 99   | 116 | 121 | 119 | 134  | 141 |

- (a) Write down
- (i) the mean temperature;
  - (ii) the standard deviation of the temperatures. [2 marks]
- (b) Write down the correlation coefficient,  $r$ , for the variables  $n$  and  $T$ . [1 mark]
- (c) Comment on your value for  $r$ . [2 marks]
- (d) The equation of the line of regression for  $n$  on  $T$  is  $n = dT - 100$ .
- (i) Write down the value of  $d$ .
  - (ii) Estimate how many bottles of water will be sold when the temperature is  $19.6^{\circ}$ . [2 marks]
- (e) On a day when the temperature was  $36^{\circ}$  Peter calculates that 314 bottles would be sold. Give one reason why his answer might be unreliable. [1 mark]

## ANSWERS

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6. (a) symmetry of normal curve (M1)  
 e.g.  $P(X < 25) = 0.5$   
 $P(X > 27) = 0.2$  A1 N2  
[2 marks]
- (b) **METHOD 1**  
 finding standardized value (A1)  
 e.g.  $\frac{27 - 25}{\sigma}$   
 evidence of complement (M1)  
 e.g.  $1 - p$ ,  $P(X < 27)$ , 0.8  
 finding z-score (A1)  
 e.g.  $z = 0.84 \dots$   
 attempt to set up equation involving the standardized value M1  
 e.g.  $0.84 = \frac{27 - 25}{\sigma}$ ,  $0.84 = \frac{X - \mu}{\sigma}$   
 $\sigma = 2.38$  A1 N3  
[5 marks]

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9. (a) three correct pairs A1A1A1 N3  
 e.g. (2, 4), (3, 3), (4, 2), R2G4, R3G3, R4G2  
[3 marks]
- (b)  $p = \frac{1}{16}$ ,  $q = \frac{2}{16}$ ,  $r = \frac{2}{16}$  A1A1A1 N3  
[3 marks]
- (c) let  $X$  be the number of times the sum of the dice is 5  
 evidence of valid approach (M1)  
 e.g.  $X \sim B(n, p)$ , tree diagram, 5 sets of outcomes produce a win  
 one correct parameter (A1)  
 e.g.  $n = 4$ ,  $p = 0.25$ ,  $q = 0.75$   
 Fred wins prize is  $P(X \geq 3)$  (A1)  
 appropriate approach to find probability M1  
 e.g. complement, summing probabilities, using a CDF function  
 correct substitution (A1)  
 e.g.  $1 - 0.949 \dots$ ,  $1 - \frac{243}{256}$ ,  $0.046875 + 0.00390625$ ,  $\frac{12}{256} + \frac{1}{256}$   
 probability of winning =  $0.0508 \left( \frac{13}{256} \right)$  A1 N3  
[6 marks]  
Total [12 marks]

4. (a)  $\sigma = 3$  (A1)

evidence of attempt to find  $P(X \leq 24.5)$  (M1)

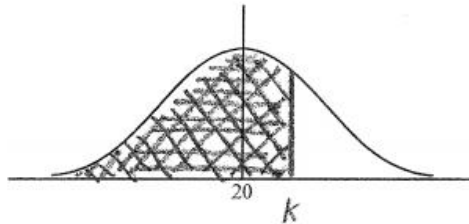
e.g.  $z = 1.5, \frac{24.5 - 20}{3}$

$P(X \leq 24.5) = 0.933$

A1 N3

[3 marks]

(b) (i)



A1A1

N2

**Note:** Award A1 with shading that clearly extends to right of the mean,  
A1 for any correct label, either  $k$ , area or their value of  $k$ .

(ii)  $z = 1.03(64338)$  (A1)

attempt to set up an equation (M1)

e.g.  $\frac{k - 20}{3} = 1.0364, \frac{k - 20}{3} = 0.85$

$k = 23.1$

A1 N3

[5 marks]

Total [8 marks]

5. (a) correct substitution into formula for  $E(X)$  (A1)

e.g.  $0.05 \times 240$

$E(X) = 12$

A1 N2

[2 marks]

(b) evidence of recognizing binomial probability (may be seen in part (a)) (M1)

e.g.  $\binom{240}{15} (0.05)^{15} (0.95)^{225}, X \sim B(240, 0.05)$

$P(X = 15) = 0.0733$

A1 N2

[2 marks]

(c)  $P(X \leq 9) = 0.236$  (A1)

evidence of valid approach (M1)

e.g. using complement, summing probabilities

$P(X \geq 10) = 0.764$

A1 N3

[3 marks]

Total [7 marks]

7. (a) evidence of valid approach involving  $A$  and  $B$  (MI)  
 e.g.  $P(A \cap \text{pass}) + P(B \cap \text{pass})$ , tree diagram
- correct expression (AI)  
 e.g.  $P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9$   
 $P(\text{pass}) = 0.84$  AI N2  
 [3 marks]
- (b) evidence of recognizing complement (seen anywhere) (MI)  
 e.g.  $P(B) = x$ ,  $P(A) = 1 - x$ ,  $1 - P(B)$ ,  $100 - x$ ,  $x + y = 1$
- evidence of valid approach (MI)  
 e.g.  $0.8(1 - x) + 0.9x$ ,  $0.8x + 0.9y$
- correct expression AI  
 e.g.  $0.87 = 0.8(1 - x) + 0.9x$ ,  $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$ ,  $0.8x + 0.9y = 0.87$
- 70 % from B AI N2  
 [4 marks]
- Total [7 marks]

7. (a) evidence of recognizing binomial distribution (MI)  
 e.g.  $X \sim B(10, 0.57)$ ,  $p = 0.57$ ,  $q = 0.43$
- EITHER
- $P(X \leq 3) = 2.16 \times 10^{-4} + 0.00286 + 0.01709 + 0.06041$  ( $= 0.08057 \dots$ ) (AI)  
 evidence of using complement (MI)  
 e.g. 1 – any probability,  $P(X \geq 4) = 1 - P(X \leq 3)$
- 0.919423...  
 $P(X \geq 4) = 0.919$  AI N3
- OR
- summing the probabilities from  $X = 4$  to  $X = 10$  (MI)  
 correct expression or values (AI)  
 e.g.  $\sum_{r=4}^{10} \binom{10}{r} (0.57)^r (0.43)^{10-r}$ ,  $0.14013 + 0.2229 + \dots + 0.02731 + 0.00362$
- 0.919424  
 $P(X \geq 4) = 0.919$  AI N3  
 [4 marks]
- (b) evidence of valid approach (MI)  
 e.g. three tails in nine tosses,  $\binom{9}{3} (0.57)^3 (0.43)^6$
- correct calculation (AI)  
 e.g.  $\binom{9}{3} (0.57)^3 (0.43)^6 \times 0.57$ ,  $0.09834 \times 0.57$
- 0.05605178...  
 $P(4^{\text{th}} \text{ tail on } 10^{\text{th}} \text{ toss}) = 0.0561$  AI N2  
 [3 marks]

# SECTION B

8. (a) (i)  $p=17, q=11$  A1A1 N2
- (ii)  $75 \leq T < 85$  A1 N1  
[3 marks]
- (b) evidence of valid approach (M1)  
e.g. adding frequencies  
 $\frac{76}{93} = 0.8172043\dots$   
 $P(T < 95) = \frac{76}{93} = 0.817$  A1 N2  
[2 marks]
- (c) (i) 10 A1 N1
- (ii) 50 A1 N1  
[2 marks]
- (d) (i) evidence of approach using mid-interval values (may be seen in part (ii)) (M1)  
 $79.1397849$   
 $\bar{x} = 79.1$  A2 N3
- (ii)  $16.4386061$   
 $\sigma = 16.4$  A1 N1  
[4 marks]
- (e) evidence of valid approach (M1)  
e.g. standardizing,  $z = 0.9648\dots$   
 $0.8326812$   
 $P(T < 95) = 0.833$  A1 N2  
[2 marks]
4. (a) evidence of appropriate method (M1)  
e.g.  $z = \frac{122.5 - 117}{5}$ , sketch of normal curve showing mean and 122.5, 1.1  
 $P(Z < 1.1) = 0.8643$  (A1)  
 $0.135666$   
 $P(H > 122.5) = 0.136$  A1 N3  
[3 marks]
- (b)  $z = 0.3853$  (A1)  
set up equation (M1)  
e.g.  $\frac{X - 117}{5} = 0.3853$ , sketch  
 $k = 118.926602$   
 $k = 199$  A1 N3  
[3 marks]
- Total [6 marks]

7.

- (a) evidence of recognizing binomial (seen anywhere)  
e.g.  $B(n, p)$ ,  $0.95^{30}$

(M1)

finding  $P(X=0)=0.21463876$

(A1)

appropriate approach  
e.g. complement, summing probabilities

(M1)

0.785361  
probability is 0.785

A1 N3  
[4 marks]

- (b) identifying correct outcomes (seen anywhere)  
e.g.  $P(X=1)+P(X=2)$ , 1 or 2 defective,  $0.3389\dots+0.2586\dots$

(A1)

recognizing conditional probability (seen anywhere)  
e.g.  $P(A|B)$ ,  $P(X \leq 2|X \geq 1)$ ,  $P(\text{at most 2}|\text{at least 1})$

R1

appropriate approach involving conditional probability  
e.g.  $\frac{P(X=1)+P(X=2)}{P(X \geq 1)}$ ,  $\frac{0.3389\dots+0.2586\dots}{0.785\dots}$ ,  $\frac{1 \text{ or } 2}{0.785}$

(M1)

0.760847  
probability is 0.761

A1 N2  
[4 marks]

Total [8 marks]

|           |  |     |  |                  |                         |
|-----------|--|-----|--|------------------|-------------------------|
| <b>Q3</b> | (i)  | (a) | (i) $a = 19$   | (A1)             | [2 marks]               |
|           |  |     | (ii) $b = 2$   | (A1)             |                         |
|           |  | (b) | $c = \frac{360}{12}$   | (M1)             |                         |
|           | <b>Or</b>  |     |  |                  |                         |
|           | For substituting a point from the graph in the equation. |     |  | (M1)             | [1 mark]                |
|           | $c = 30$   |     |  | (AG)             |                         |
|           |  |     |  |                  |                         |
|           | (ii)   | (a) | (i) $3.5 < x < 8.5$  | (A1)(A1)         | [2 marks]               |
|           |  |     | (A1) for 3.5 and 8.5 seen, (A1) for strict inequality signs seen.<br>Accept $3.48 < x < 8.52$ (found from GDC) |                  |                         |
|           |  | (a) | (i) 19.2   | (G1)             | [2 marks]               |
|           |  |     | (ii) 1.45  | (G1)             |                         |
|           |  | (b) | $r = 0.942$  | (G1)             | [1 mark]                |
|           |  | (c) | Strong, positive correlation.  | (A1)(ft)(A1)(ft) | [2 marks]               |
|           |  | (d) | (i) $d = 11.5$   | (G1)             | [2 marks]               |
|           |  |     | (ii) $n = 11.5 \times 19.6 - 100$<br>$= 125$ (accept 126)<br>Answer must be a whole number                     | (A1)(ft)         |                         |
|           |  |     |  | (R1)             |                         |
|           |  | (e) | It is unreliable to extrapolate outside the values given (outlier).  |                  | [1 mark]                |
|           |  |     |  |                  | <b>Total [13 marks]</b> |