HILLEL ACADEMY HIGH
MATHEMATICS DEPARTMENT
SEQUENCES AND SERIES

NAME: _____ DATE: _____

Arithmetic Sequences and Series:

EXERCISE SET 11.2

A Find the first term and the common difference.

1. $3, 8, 13, 18, \ldots$
2. $1.08, 1.16, 1.24, 1.32, \ldots$
3. $9, 5, 1, -3, \ldots$
4. $-8, -5, -2, 1, 4, \ldots$
5. $\frac{3}{2}, \frac{4}{9}, 3, \frac{15}{4}, \ldots$
6. $\frac{3}{5}, \frac{1}{10}, -\frac{2}{5}, \ldots$
7. $\$1.07, \$1.14, \$1.21, \$1.28, \ldots$
8. $\$316, \$313, \$310, \$307, \ldots$
9. Find the 12th term of the arithmetic sequence $2, 6, 10, \ldots$.
10. Find the 11th term of the arithmetic sequence $0.07, 0.12, 0.17, \ldots$.
11. Find the 17th term of the arithmetic sequence $7, 4, 1, \ldots$.
12. Find the 14th term of the arithmetic sequence $3, \frac{7}{3}, \frac{5}{3}, \ldots$.
13. Find the 13th term of the arithmetic sequence $\$1200, \$964.32, \$728.64, \ldots$.
14. Find the 10th term of the arithmetic sequence $\$2345.78, \$2967.54, \$3589.30, \ldots$.
15. In the sequence of Exercise 9, what term is 106?
16. In the sequence of Exercise 10, what term is 1.67?
17. In the sequence of Exercise 11, what term is -296?
18. In the sequence of Exercise 12, what term is -27?
19. Find a_{17} when $a_1 = 5$ and $d = 6$.
20. Find a_{20} when $a_1 = 14$ and $d = -3$.
21. Find a_1 when $d = 4$ and $a_8 = 33$.
22. Find d when $a_1 = 8$ and $a_{11} = 26$.
23. Find n when $a_1 = 5$, $d = -3$, and $a_n = -76$.
24. Find n when $a_1 = 25$, $d = -14$, and $a_n = -507$.
25. In an arithmetic sequence, $a_{17} = -40$ and $a_{28} = -73$. Find a_1 and d. Write the first 5 terms of the sequence.
26. In an arithmetic sequence, $a_{17} = \frac{25}{3}$ and $a_{32} = \frac{95}{6}$. Find a_1 and d. Write the first 5 terms of the sequence.

B

27. Find the sum of the first 20 terms of the series $5 + 8 + 11 + 14 + \cdots$.
28. Find the sum of the first 14 terms of the series $11 + 7 + 3 + \cdots$.
29. Find the sum of the first 300 natural numbers.
30. Find the sum of the first 400 natural numbers.
31. Find the sum of the even numbers from 2 to 100, inclusive.
32. Find the sum of the odd numbers from 1 to 99, inclusive.
33. Find the sum of the multiples of 7 from 7 to 98, inclusive.
34. Find the sum of all multiples of 4 that are between 14 and 523.
35. If an arithmetic series has $a_1 = 2$, $d = 5$, and $n = 20$, what is S_n?
36. If an arithmetic series has $a_1 = 7$, $d = -3$, and $n = 32$, what is S_n?

C

37. A gardener is making a triangular planting, with 35 plants in the front row, 31 in the second row, 27 in the third row, and so on. If the pattern is consistent, how many plants will there be in the last row?
38. A formation of a marching band has 14 marchers in the front row, 16 in the second row, 18 in the third row, and so on, for 25 rows. How many marchers are in the last row? How many marchers are there altogether?
39. How many poles will be in a pile of telephone poles if there are 50 in the first layer, 49 in the second, and so on, until there is 1 in the last layer?
40. If 10¢ is saved on October 1, 20¢ on October 2, 30¢ on October 3, and so on, how much is saved during October? (October has 31 days.)
41. A family saves money in an arithmetic sequence. They save \$600 the first year, \$700 the second, and so on, for 20 years. How much do they save in all (disregarding interest)?
42. A student saves \$30 on August 1, \$50 on August 2, \$70 on August 3, and so on. How much will she save in August? August has 31 days.

Geometric Sequences and Series:

7. $75, 15, 3, \frac{3}{5}, \ldots$
8. $6.275, 0.6275, 0.06275, \ldots$
9. $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \ldots$
10. $5, \frac{5m}{2}, \frac{5m^2}{4}, \frac{5m^3}{8}, \ldots$
11. \$780, \$858, \$943.80, \$1038.18, . . .
12. \$5600, \$5320, \$5054, \$4801.30, . . .

Find the indicated term.

13. $2, 4, 8, 16, \ldots$; the 6th term
14. $2, -10, 50, -250, \ldots$; the 9th term
15. $2, 2\sqrt{3}, 6, \ldots$; the 9th term
16. $1, -1, 1, -1, \ldots$; the 57th term
17. $\frac{8}{243}, \frac{8}{81}, \frac{8}{27}, \ldots$; the 10th term
18. $\frac{7}{625}, \frac{-7}{25}, \ldots$; the 23rd term
19. \$1000, \$1080, \$1166.40, . . . ; the 5th term
20. \$1000, \$1070, \$1144.90 , . . . ; the 6th term

Find the nth, or general, term.

21. $1, 3, 9, \ldots$
22. $25, 5, 1, \ldots$
23. $1, -1, 1, -1, \ldots$
24. $2, 4, 8, \ldots$
25. $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \ldots$
26. $5, \frac{5m}{2}, \frac{5m^2}{4}, \ldots$

B

27. Find the sum of the first 7 terms of the geometric series
$$6 + 12 + 24 + \cdots.$$
28. Find the sum of the first 6 terms of the geometric series
$$16 - 8 + 4 - \cdots.$$
29. Find the sum of the first 7 terms of the geometric series
$$\frac{1}{18} - \frac{1}{6} + \frac{1}{2} - \cdots.$$
30. Find the sum of the geometric series
$$-8 + 4 + (-2) + \cdots + \left(-\frac{1}{32}\right).$$
31. Find the sum of the first 8 terms of the series
$$1 + x + x^2 + x^3 + \cdots.$$
32. Find the sum of the first 10 terms of the series
$$1 + x^2 + x^4 + x^6 + \cdots.$$
33. Find the sum of the first 16 terms of the geometric sequence
$$\$200, \$200(1.06), \$200(1.06)^2, \ldots.$$
34. Find the sum of the first 23 terms of the geometric sequence
$$\$1000, \$1000(1.08), \$1000(1.08)^2, \ldots.$$

C

35. Find the sum:
$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1}.$$
36. Find the sum:
$$\sum_{k=1}^{\infty} 2^k.$$

Determine whether each of the following infinite geometric series has a sum. If so, find it.

37. $4 + 2 + 1 + \cdots$
38. $7 + 3 + \frac{9}{7} + \cdots$
39. $25 + 20 + 16 + \cdots$
40. $12 + 9 + \frac{27}{4} + \cdots$
41. $100 - 10 + 1 - \frac{1}{10} + \cdots$
42. $-6 + 18 - 54 + 162 - \cdots$
43. $8 + 40 + 200 + \cdots$
44. $-6 + 3 - \frac{3}{2} + \frac{3}{4} - \cdots$
45. $0.6 + 0.06 + 0.006 + \cdots$
46. $0.37 + 0.0037 + 0.000037 + \cdots$
47. $\$500(1.11)^{-1} + \$500(1.11)^{-2} + \$500(1.11)^{-3} + \cdots$
48. $\$1000(1.08)^{-1} + \$1000(1.08)^{-2} + \$1000(1.08)^{-3} + \cdots$
49. $\sum_{k=1}^{\infty} 16(0.1)^{k-1}$
50. $\sum_{k=1}^{\infty} 4(0.6)^{k-1}$
51. $\sum_{k=1}^{\infty} \frac{1}{2^{k-1}}$
52. $\sum_{k=1}^{\infty} \frac{8}{3}\left(\frac{1}{2}\right)^{k-1}$

Find fractional notation for each of the following infinite sums. (These are geometric series.)

53. $0.777\overline{7}$
54. $8.999\overline{9}$
55. $0.533\overline{3}$
56. $0.644\overline{4}$
57. $5.1515\overline{15}$
58. $0.4125\overline{125}$

D **Problem Solving**

59. A ping-pong ball is dropped from a height of 16 ft and always rebounds $\frac{1}{4}$ of the distance fallen. How high does it rebound the 6th time?
60. Approximate the total amount of the rebound heights of the ball in Exercise 59.
61. Gaintown has a population of 100,000 now, and the population is increasing by 3% each year. What will the population be in 15 years?
62. How long will it take for the population of Gaintown to double? (See Exercise 61.)
63. A student borrows \$1200. The loan is to be repaid in 13 years at 12% interest, compounded annually. How much will be repaid at the end of 13 years?
64. A piece of paper is 0.01 in. thick. It is folded repeatedly in such a way that its thickness is doubled each time for 20 times. How thick is the result?
65. A superball dropped from the top of the Washington Monument (556 ft high) always rebounds $\frac{3}{4}$ of the distance fallen. How far (up and down) will the ball have traveled when it hits the ground for the 6th time?
66. Approximate the total distance that the ball of Exercise 65 will have traveled when it comes to rest.