Hillel Academy

Grade 12

Mathematics Study Guide

September 2012 – June 2013

Examination	Month
The exam consists of *2 papers*: Paper 1: *(1 Hour and 30 Minutes)* ***non-calculator*** Paper 2: *(1 Hour and 30 Minutes)* ***calculator*** ***Required material*** *: pen, pencil, calculator and ruler*	June

GRADE 12 MATHEMATICS

GRADES

- The End of Year Report has 6 columns:

P1	P2	P3	P4	Exam Grade	Year Grade

- Students scoring 86% and above in the Year grade will be awarded 1st Honours.
- Students scoring 76% and above in the Year grade will be awarded 2nd Honours.
- The pass mark is 46%

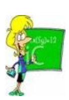
HOW TO STUDY

- ✓ Start now! Do not wait until exam week!

- ✓ Plan your study time systematically. Set aside at least 1 hour every day to revise mathematics. On the days that you do Math try and put in at least 3 hours.

- ✓ Read your notes and your textbook and practise some of the 'Review Exercises' at the end of each chapter. Check your answers at the back of the book.
- ✓ Make sure that you understand and can apply all mathematical vocabulary.

- ✓ Remember that the showing working is very important. Many of the questions will be worth more than one mark. You will only be awarded full marks if all your work is set out clearly.



- ✓ Use your own exercise book(s) to help you revise. Go over your own work. Look at the mistakes you have made. Do you know how to do this work correctly now? <u>If not, ask your teacher for help.</u>

Outline of Content to be covered for End of Year Examination

ALGEBRA

	Content	Further guidance
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Technology may be used to generate and display sequences in several ways. Link to 2.6, exponential functions. Examples include compound interest and population growth.

	Content	Further guidance
1.2	Elementary treatment of exponents and logarithms.	*Examples*: $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$; $\log 32 = 5 \log 2$; $(2^3)^{-4} = 2^{-12}$.
	Laws of exponents; laws of logarithms. Change of base.	*Examples*: $\log_4 7 = \frac{\ln 7}{\ln 4}$, $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} \left(= \frac{3}{2} \right)$. Link to 2.6, logarithmic functions.
1.3	The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$. **Not required:** formal treatment of permutations and formula for ${}^nP_r$.	Counting principles may be used in the development of the theorem. $\binom{n}{r}$ should be found using **both** the formula and technology. *Example*: finding $\binom{6}{r}$ from inputting $y = 6^nC_rX$ and then reading coefficients from the table. Link to 5.8, binomial distribution.

FUNCTIONS AND EQUATIONS

	Content	Further guidance
2.1	Concept of function $f: x \mapsto f(x)$. Domain, range; image (value).	*Example*: for $x \mapsto \sqrt{2-x}$, domain is $x \le 2$, range is $y \ge 0$. A graph is helpful in visualizing the range.
	Composite functions.	$(f \circ g)(x) = f(g(x))$.
	Identity function. Inverse function f^{-1}.	$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.
	Not required: domain restriction.	On examination papers, students will only be asked to find the inverse of a *one-to-one* functio[n]
2.2	The graph of a function; its equation $y = f(x)$.	
	Function graphing skills.	**Note** the difference in the command terms "draw" and "sketch".
	Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.	
	Use of technology to graph a variety of functions, including ones not specifically mentioned.	An analytic approach is also expected for simple functions, including all those listed under topic 2.
	The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.	Link to 6.3, local maximum and minimum points.
	Content	**Further guidance**
2.3	Transformations of graphs.	Technology should be used to investigate these transformations.
	Translations: $y = f(x) + b$; $y = f(x-a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f(qx)$.	Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down.
	Composite transformations.	*Example*: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.
2.4	The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry. The form $x \mapsto a(x-p)(x-q)$, x-intercepts $(p, 0)$ and $(q, 0)$. The form $x \mapsto a(x-h)^2 + k$, vertex (h, k).	Candidates are expected to be able to change from one form to another. Links to 2.3, transformations; 2.7, quadratic equations.

FUNCTIONS AND EQUATIONS

	Content	Further guidance
2.5	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.	
	The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.	*Examples*: $h(x) = \frac{4}{3x-2}$, $x \neq \frac{2}{3}$; $y = \frac{x+7}{2x-5}$, $x \neq \frac{5}{2}$.
	Vertical and horizontal asymptotes.	Diagrams should include all asymptotes and intercepts.
2.6	Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$.	
	Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$. Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.	Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits.

	Content	Further guidance
2.7	Solving equations, both graphically and analytically.	Solutions may be referred to as roots of equations or zeros of functions.
	Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.	Links to 2.2, function graphing skills; and 2.3–2.6, equations involving specific functions. *Examples*: $e^x = \sin x$, $x^4 + 5x - 6 = 0$.
	Solving $ax^2 + bx + c = 0$, $a \neq 0$. The quadratic formula.	
	The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.	*Example*: Find k given that the equation $3kx^2 + 2x + k = 0$ has two equal real roots.
	Solving exponential equations.	*Examples*: $2^{x-1} = 10$, $\left(\frac{1}{3}\right)^x = 9^{x+1}$. Link to 1.2, exponents and logarithms.
2.8	Applications of graphing skills and solving equations that relate to real-life situations.	Link to 1.1, geometric series.

Circular Functions and Trigonometry

	Content	Further guidance
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π, or decimals.
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.	The equation of a straight line through the origin is $y = x\tan\theta$. *Examples*: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan 210° = \frac{\sqrt{3}}{3}$.

	Content	Further guidance
3.3	The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine. Relationship between trigonometric ratios.	Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities). *Examples*: Given $\sin\theta$, finding possible values of $\tan\theta$ without finding θ. Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x.
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs. Composite functions of the form $f(x) = a\sin(b(x+c)) + d$. Transformations. Applications.	*Examples*: $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2\cos(3(x-4)) + 1$. *Example*: $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction. Link to 2.3, transformation of graphs. Examples include height of tide, motion of a Ferris wheel.

Circular Functions and Trigonometry

	Content	Further guidance
3.5	Solving trigonometric equations in a finite interval, both graphically and analytically.	*Examples*: $2\sin x = 1$, $0 \le x \le 2\pi$, $2\sin 2x = 3\cos x$, $0° \le x \le 180°$, $2\tan(3(x-4)) = 1$, $-\pi \le x \le 3\pi$.
	Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$. **Not required:** the general solution of trigonometric equations.	*Examples*: $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \le x < 4\pi$, $2\sin x = \cos 2x$, $-\pi \le x \le \pi$.
3.6	Solution of triangles.	Pythagoras' theorem is a special case of the cosine rule.
	The cosine rule. The sine rule, including the ambiguous case. Area of a triangle, $\frac{1}{2}ab\sin C$.	Link with 4.2, scalar product, noting that: $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b} \Rightarrow \lvert\boldsymbol{c}\rvert^2 = \lvert\boldsymbol{a}\rvert^2 + \lvert\boldsymbol{b}\rvert^2 - 2\boldsymbol{a}\cdot\boldsymbol{b}$.
	Applications.	Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.

Statistics and Probability

	Content	Further guidance
5.1	Concepts of population, sample, random sample, discrete and continuous data.	Continuous and discrete data.
	Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals;	
	box-and-whisker plots; outliers.	Outlier is defined as more than $1.5 \times \text{IQR}$ from the nearest quartile. Technology may be used to produce histograms and box-and-whisker plots.
	Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class. **Not required:** frequency density histograms.	

	Content	Further guidance
5.2	Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles.	On examination papers, data will be treated as the population. Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data.
	Dispersion: range, interquartile range, variance, standard deviation.	Calculation of standard deviation/variance using only technology.
	Effect of constant changes to the original data.	Link to 2.3, transformations. *Examples*: If 5 is subtracted from all the data items, then the mean is decreased by 5, but the standard deviation is unchanged. If all the data items are doubled, the median is doubled, but the variance is increased by a factor of 4.
	Applications.	
5.3	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.	Values of the median and quartiles produced by technology may be different from those obtained from a cumulative frequency graph.

Statistics and Probability

	Content	Further guidance
5.4	Linear correlation of bivariate data.	Independent variable x, dependent variable y.
	Pearson's product–moment correlation coefficient r.	Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Positive, zero, negative; strong, weak, no correlation.
	Scatter diagrams; lines of best fit.	The line of best fit passes through the mean point.
	Equation of the regression line of y on x.	Technology should be used find the equation.
	Use of the equation for prediction purposes.	Interpolation, extrapolation.
	Mathematical and contextual interpretation.	
	Not required: the coefficient of determination R^2.	
5.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event.	The sample space can be represented diagrammatically in many ways.
	The probability of an event A is $\mathrm{P}(A) = \frac{n(A)}{n(U)}$.	Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between (experimental) relative frequency and (theoretical) probability.
	The complementary events A and A' (not A).	Simulations may be used to enhance this topic.
	Use of Venn diagrams, tree diagrams and tables of outcomes.	Links to 5.1, frequency; 5.3, cumulative frequency.

	Content	Further guidance
5.6	Combined events, $\mathrm{P}(A \cup B)$.	The non-exclusivity of "or".
	Mutually exclusive events: $\mathrm{P}(A \cap B) = 0$.	Problems are often best solved with the aid of a Venn diagram or tree diagram, without explicit use of formulae.
	Conditional probability; the definition $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$.	
	Independent events; the definition $\mathrm{P}(A \mid B) = \mathrm{P}(A) = \mathrm{P}(A \mid B')$.	
	Probabilities with and without replacement.	