**Vectors worksheet ans**

**IB Level 12 – 13**

**1.** $\boldsymbol{u} + \boldsymbol{v} = 4\boldsymbol{i} + 3\boldsymbol{j}$ (A1)

Then $a(4\boldsymbol{i} + 3\boldsymbol{j}) = 8\boldsymbol{i} + (b - 2)\boldsymbol{j}$

$4a = 8$

$3a = b - 2$ (A1)

Whence $a = 2$ (A1) (C2)

$b = 8$ (A1) (C2)

**[4]**

**2.** (i) $|\boldsymbol{a}| = \sqrt{12^2 + 5^2} = 13$ (A1)

(ii) $|\boldsymbol{b}| = \sqrt{6^2 + 8^2} = 10$ (A1)

=> unit vector in direction of $\boldsymbol{b} = \frac{1}{10}(6\boldsymbol{i} + 8\boldsymbol{j})$ (A1)

$= 0.6\boldsymbol{i} + 0.8\boldsymbol{j}$

(iii) $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos \theta$ (M1)

=> $\cos \theta = \frac{12(6) + 5(8)}{13(10)}$ (A1)

$= \frac{112}{130} = \frac{56}{65}$ (A1) 6

**[6]**

**3.** Angle between lines = angle between direction vectors. (M1)

Direction vectors are $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **and** $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. (A1)

$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| \cos \theta$ (M1)

$4(1) + 3(-1) = \left(\sqrt{4^2 + 3^2}\right)\left(\sqrt{1^2 + (-1)^2}\right) \cos \theta$ (A1)

$\cos \theta = \frac{1}{5\sqrt{2}} = 0.1414$ (A1)

$\theta = 81.9°$ (3 sf), (1.43 radians) (A1) (C6)

*Note:* *If candidates find the angle between the vectors $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, award marks as below:*

Angle required is between $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (M0)(A0)

$\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| \left|\begin{pmatrix} 2 \\ 4 \end{pmatrix}\right| \cos\theta$ (M1)

$4(2) + (-1)\,4 = \left(\sqrt{4^2 + (-1)^2}\right)\left(\sqrt{2^2 + 4^2}\right) \cos\theta$ (A1)

$\dfrac{4}{\sqrt{17}\sqrt{20}} = \cos\theta = 0.2169$ (A1)

$\theta = 77.5°$ (3sf), (1.35 radians) (A1) (C4)

**[6]**

**4.** Required vector will be parallel to $\begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ (A1)

Hence required equation is $\boldsymbol{r} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ (A1)(A1) (C4)

***Note:*** *Accept alternative answers, eg* $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

**[4]**

**5.** $\begin{pmatrix} 2 \\ 3 \end{pmatrix} . \begin{pmatrix} x-4 \\ y+1 \end{pmatrix}$ (M1) (M1)

***Notes:*** *Award (M1) for using scalar product.*
*Award (M1) for* $\begin{pmatrix} x-4 \\ y+1 \end{pmatrix}$.

$2(x - 4) + 3(y + 1) = 0$ (A1)

$2x - 8 + 3y + 3 = 0$

$2x + 3y = 5$ (A1)

**OR**

Gradient of a line parallel to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is $\frac{3}{2}$ (M1)

Gradient of a line perpendicular to this line is $-\frac{2}{3}$ (M1)

So the equation is $y + 1 = -\frac{2}{3}(x - 4)$ (A1)

$\Rightarrow 3y + 3 = -2x + 8$

$\Rightarrow 2x + 3y = 5$ (A1)

**[4]**

**6.** Direction vectors are $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j}$. (A2)

$\boldsymbol{a} \cdot \boldsymbol{b} = (1 + 3)$ (A1)

$|\boldsymbol{a}| = \sqrt{10}$, $|\boldsymbol{b}| = \sqrt{2}$ (A1)

$\cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\left(= \frac{4}{\sqrt{10}\sqrt{2}}\right)$ (M1)

$\cos\theta = \frac{4}{\sqrt{20}}$ (A1) (C6)

**[6]**

**7.** (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ (M1)

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ (A1) (N2) 2

(ii) $\left|\overrightarrow{AB}\right| = \sqrt{25+1}$ (M1)

$= \sqrt{26}$ (= 5.10 to 3 sf) (A1) (N2) 2

***Note:*** *An answer of 5.1 is subject to* ***AP****.*

(b) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix} d \\ 23 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} d-2 \\ 25 \end{pmatrix}$ (A1)(A1) 2

(c) (i) **EITHER**

$\mathrm{B\hat{A}D} = 90° \Rightarrow \overrightarrow{AB} \bullet \overrightarrow{AD} = 0$ or mention of scalar (dot) product. (M1)

$\Rightarrow \begin{pmatrix}-5\\1\end{pmatrix} \bullet \begin{pmatrix}d-2\\25\end{pmatrix} = 0$

$-5d + 10 + 25 = 0$ (A1)

$d = 7$ (AG)

**OR**

$\left.\begin{array}{l}\text{Gradient of AB} = -\dfrac{1}{5}\\ \text{Gradient of AD} = \dfrac{25}{d-2}\end{array}\right\}$ (A1)

$\left(\dfrac{25}{d-2}\right) \times \left(-\dfrac{1}{5}\right) = -1$ (A1)

$\Rightarrow d = 7$ (AG)

(ii) $\overrightarrow{OD} = \begin{pmatrix}7\\23\end{pmatrix}$ (correct answer only) (A1) 3

(d) $\overrightarrow{AD} = \overrightarrow{BC}$ (M1)

$\overrightarrow{BC} = \begin{pmatrix}5\\25\end{pmatrix}$ (A1)

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ (M1)

$\overrightarrow{OC} = \begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}5\\25\end{pmatrix}$

$= \begin{pmatrix}2\\24\end{pmatrix}$ (A1) (N3) 4

***Note:*** *Many other methods, including scale drawing, are acceptable.*

(e) $\left|\overrightarrow{AD}\right| \left(\text{or } \left|\overrightarrow{BC}\right|\right) = \sqrt{5^2 + 25^2} = \sqrt{650}$ (A1)

Area = $\sqrt{26} \times \sqrt{650}$ =( 5.099 × 25.5)

= 130 (A1) 2

**[15]**

**8.** (a) $\left|\overrightarrow{OA}\right| = 6 \quad \Rightarrow$ *A* is on the circle (A1)

$\left|\overrightarrow{OB}\right| = 6 \quad \Rightarrow$ *B* is on the circle. (A1)

$$\left|\overrightarrow{OC}\right| = \left|\begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix}\right|$$

$= \sqrt{25+11}$

$= 6 \quad \Rightarrow$ *C* is on the circle. (A1) 3

(b) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$$= \begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix} \qquad \text{(M1)}$$

$$= \begin{pmatrix} -1 \\ \sqrt{11} \end{pmatrix} \qquad \text{(A1)} \quad 2$$

(c)

$$\cos O\hat{A}C = \frac{\overrightarrow{AO} \cdot \overrightarrow{AC}}{\left|AO\right|\left|AC\right|} \qquad \text{(M1)}$$

$$= \frac{\begin{pmatrix} -6 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ \sqrt{11} \end{pmatrix}}{6\sqrt{1+11}}$$

$$= \frac{6}{6\sqrt{12}} \qquad \text{(A1)}$$

$$= \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6} \qquad \text{(A1)}$$

**OR** $\cos O\hat{A}C = \dfrac{6^2 + (\sqrt{12})^2 - 6^2}{2 \times 6 \times \sqrt{12}}$ (M1)(A1)

$= \dfrac{1}{\sqrt{12}}$ as before (A1)

**OR** using the triangle formed by $\overrightarrow{AC}$ and its horizontal and vertical components:

$\left|\overrightarrow{AC}\right| = \sqrt{12}$ (A1)

$\cos O\hat{A}C = \dfrac{1}{\sqrt{12}}$ (M1)(A1) 3

***Note:*** *The answer is 0.289 to 3 sf*

(d) A number of possible methods here

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix} - \begin{pmatrix} -6 \\ 0 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 11 \\ \sqrt{11} \end{pmatrix}$ (A1)

$| BC | = \sqrt{132}$

$| \Delta ABC | = \frac{1}{2} \times \sqrt{132} \times \sqrt{12}$ (A1)

$= 6\sqrt{11}$ (A1)

**OR** $\Delta ABC$ has base $AB$ = 12 (A1)

and height = $\sqrt{11}$ (A1)

$\Rightarrow$ area = $\frac{1}{2} \times 12 \times \sqrt{11}$ (A1)

$= 6\sqrt{11}$ (A1)

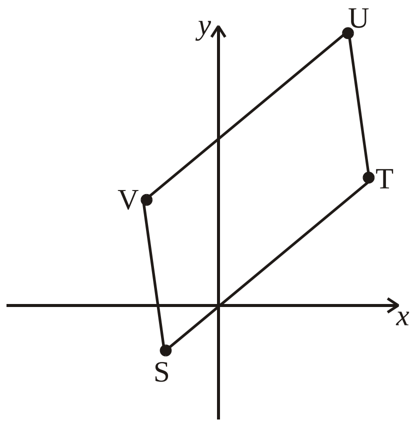
**OR** Given $\cos B\hat{A}C = \frac{\sqrt{3}}{6}$

$\sin B\hat{A}C = \frac{\sqrt{33}}{6} \Rightarrow |\Delta ABC| = \frac{1}{2} \times 12 \times \sqrt{12} \times \frac{\sqrt{33}}{6}$ (A1)(A1)(A1)

$= 6\sqrt{11}$ (A1) 4

**[12]**

**9.**




(a) $\overrightarrow{ST} = \boldsymbol{t} - \boldsymbol{s}$ (M1)

$= \begin{pmatrix} 7 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 9 \end{pmatrix}$ (A1)

$\overrightarrow{VU} = \overrightarrow{ST}$ (M1)

$\boldsymbol{u} - \boldsymbol{v} = \begin{pmatrix} 9 \\ 9 \end{pmatrix}$

$\boldsymbol{v} = \boldsymbol{u} - \begin{pmatrix} 9 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 15 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ (A1)

V(–4, 6) (A1) 5

(b) Equation of (UV): direction is $= \begin{pmatrix} 9 \\ 9 \end{pmatrix} \left( \text{or } k\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$ (A1)

$\boldsymbol{r} = \begin{pmatrix} 5 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 9 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (A1)

**OR**

$\boldsymbol{r} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 9 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (A1) 2

(c) $\begin{pmatrix}1\\11\end{pmatrix}$ is on the line because it gives the same value of $\lambda$, for both the $x$ and $y$ coordinates. (R1)

For example, $1 = 5 + 9\lambda \quad \lambda = -\frac{4}{9}$

$11 = 15 + 9\lambda \quad \lambda = -\frac{4}{9}$ (A1) 2

(d) (i) $\overrightarrow{\mathrm{EW}} = \begin{pmatrix}a\\17\end{pmatrix} - \begin{pmatrix}1\\11\end{pmatrix}$ (M1)

$= \begin{pmatrix}a-1\\6\end{pmatrix}$ (A1)

$|\overrightarrow{\mathrm{EW}}| = 2\sqrt{13} \Rightarrow \sqrt{(a-1)^2 + 36} = 2\sqrt{13}$ (or $(a-1)^2 + 36 = 52$)(M1)

$a^2 - 2a + 1 + 36 = 52$

$a^2 - 2a - 15 = 0$ (A1)

$a = 5$ or $a = -3$ (A1)(AG)

(ii) For $a = -3$

$\overrightarrow{\mathrm{EW}} = \begin{pmatrix}-4\\6\end{pmatrix}$ $\quad \overrightarrow{\mathrm{ET}} = \boldsymbol{t} - \boldsymbol{e} = \begin{pmatrix}6\\-4\end{pmatrix}$ (A1)(A1)

$\cos \mathrm{W\hat{E}T} = \dfrac{\overrightarrow{\mathrm{EW}} \cdot \overrightarrow{\mathrm{ET}}}{|\overrightarrow{\mathrm{EW}}||\overrightarrow{\mathrm{ET}}|}$ (M1)

$= \dfrac{-24 - 24}{\sqrt{52}\sqrt{52}}$ (A1)

$= -\dfrac{12}{13}$

Therefore, $\mathrm{W\hat{E}T} = 157°$ (3 sf) (A1) 10

**[19]**

**10.** (a) At 13:00, $t = 1$ (M1)

$\Rightarrow \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\28\end{pmatrix} + 1 \times \begin{pmatrix}6\\-8\end{pmatrix} = \begin{pmatrix}6\\20\end{pmatrix}$ (A1) 2

(b) (i) Velocity vector: $\begin{pmatrix} x \\ y \end{pmatrix}_{t=1} - \begin{pmatrix} x \\ y \end{pmatrix}_{t=0}$ (M1)

$= \begin{pmatrix} 6 \\ 20 \end{pmatrix} - \begin{pmatrix} 0 \\ 28 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ (km h$^{-1}$) (A1)

(ii) Speed = $\sqrt{(6^2 + (-8)^2)}$; (M1)

= 10; 10 km h$^{-1}$ (A1) 4

(c) **EITHER** $\left.\begin{array}{l} x = 6t \\ y = 28 - 8t \end{array}\right\}$ (M1)

***Note:*** *Award (M1) for both equations.*

$\Rightarrow y = 28 - 8\left(\frac{x}{6}\right)$ (M1)(A1)

***Note:*** *Award (M1) for elimination, award (A1) for equation in x, y.*

$\Rightarrow 4x + 3y = 84$ (a1) 4

**OR**

$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -8 \end{pmatrix}^{\perp} = \begin{pmatrix} 0 \\ 28 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -8 \end{pmatrix}^{\perp}$ (M1)

$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 28 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ (M1)(A1)

$\Leftrightarrow 4x + 3y = 84$ (A1) 4

(d) They collide if $\begin{pmatrix}18\\4\end{pmatrix}$ lies on path; (R1)

**EITHER** (18, 4) lies on $4x + 3y = 84$

$\Leftrightarrow 4 \times 18 + 3 \times 4 = 84$

$\Leftrightarrow 72 + 12 = 84$; OK; (M1)

$x = 18$ (M1)

$\Rightarrow 18 = 6t \Rightarrow t = 3$, collide at 15:00 (A1) 4

**OR** $\begin{pmatrix}18\\4\end{pmatrix} = \begin{pmatrix}0\\28\end{pmatrix} + t\begin{pmatrix}6\\-8\end{pmatrix}$ for some $t$,

$\Leftrightarrow \left\{\begin{matrix} 18 = 6t \\ \textbf{and}\, 4 = 28 - 8t \end{matrix}\right\}$ (A1)

$\Leftrightarrow \left\{\begin{matrix} t = 3 \\ \textbf{and}\, 8t = 24 \end{matrix}\right\}$ (A1)

$\Leftrightarrow \left\{\begin{matrix} t = 3 \\ \textbf{and}\, t = 3 \end{matrix}\right\}$

They collide at 15:00 (A1) 4

(e) $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}18\\4\end{pmatrix} + (t-1)\begin{pmatrix}5\\12\end{pmatrix}$ (M1)

$= \begin{pmatrix}18 + 5t - 5\\4 + 12t - 12\end{pmatrix}$ (M1) 2

$= \begin{pmatrix}13\\-8\end{pmatrix} + t\begin{pmatrix}5\\12\end{pmatrix}$ (AG)

(f) At $t = 3$, (M1)

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}13 + 3 \times 5\\-8 + 3 \times 12\end{pmatrix} = \begin{pmatrix}28\\28\end{pmatrix}$ (A1)

$\begin{pmatrix}28\\28\end{pmatrix} - \begin{pmatrix}18\\4\end{pmatrix} = \begin{pmatrix}10\\24\end{pmatrix}$ (A1)

$\sqrt{(10^2 + 24^2)} = \sqrt{(676)} = 26$

26 km apart (A1) 4

**[20]**

**11.** (a) $\left|\begin{pmatrix}18\\24\end{pmatrix}\right| = 30 \text{ km h}^{-1}$ (A1)

$\left|\begin{pmatrix}36\\-16\end{pmatrix}\right| = \sqrt{36^2 + (-16)^2}$

$= 39.4$ (A1) 2

(b) (i) After ½ hour, position vectors are

$\begin{pmatrix}9\\12\end{pmatrix}$ and $\begin{pmatrix}18\\-8\end{pmatrix}$ (A1)(A1)

(ii) At 6.30 am, vector joining their positions is

$\begin{pmatrix}9\\12\end{pmatrix} - \begin{pmatrix}18\\-8\end{pmatrix} = \begin{pmatrix}-9\\20\end{pmatrix}$ (or $\begin{pmatrix}9\\-20\end{pmatrix}$) (M1)

$\left|\begin{pmatrix}-9\\20\end{pmatrix}\right|$ (M1)

$= \sqrt{481}$ (= 21.9 km to 3 sf) (A1) 5

(c) The Toyundai must continue until its position vector is $\begin{pmatrix}18\\k\end{pmatrix}$ (M1)

Clearly $k = 24$, *ie* position vector $\begin{pmatrix}18\\24\end{pmatrix}$. (A1)

To reach this position, it must travel for 1 hour in total. (A1)
Hence the crew starts work at 7.00 am (A1) 4

(d) Southern (Chryssault) crew lays 800 × 5 = 4000 m (A1)
Northern (Toyundai) crew lays 800 × 4.5 = 3600 m (A1)
Total by 11.30 am = 7.6 km

Their starting points were 24 – (–8) = 32 km apart (A1)
Hence they are now 32 – 7.6 = 24.4 km apart (A1) 4

(e) Position vector of Northern crew at 11.30 am is

$\begin{pmatrix} 18 \\ 24-3.6 \end{pmatrix} = \begin{pmatrix} 18 \\ 20.4 \end{pmatrix}$ (M1)(A1)

Distance to base camp = $\left|\begin{pmatrix} 18 \\ 20.4 \end{pmatrix}\right|$ (A1)

= 27.2 km

Time to cover this distance = $\frac{27.2}{30} \times 60$ (A1)

= 54.4 minutes

= 54 minutes (to the nearest minute) (A1) 5

**[20]**

**12.** (a) (i) $\overrightarrow{OA} = \begin{pmatrix} 240 \\ 70 \end{pmatrix}$ OA = $\sqrt{240^2 + 70^2}$ = 250 (A1)

unit vector = $\frac{1}{250}\begin{pmatrix} 240 \\ 70 \end{pmatrix} = \begin{pmatrix} 0.96 \\ 0.28 \end{pmatrix}$ (M1)(AG)

(ii) $\bar{\boldsymbol{v}} = 300\begin{pmatrix} 0.96 \\ 0.28 \end{pmatrix} = \begin{pmatrix} 288 \\ 84 \end{pmatrix}$ (M1)(A1)

(iii) $t = \frac{240}{288} = \frac{5}{6}$ hr (= 50 min) (A1) 5

(b) $\overrightarrow{AB} = \begin{pmatrix} 480-240 \\ 250-70 \end{pmatrix} = \begin{pmatrix} 240 \\ 180 \end{pmatrix}$ (A1)

AB = $\sqrt{240^2 + 180^2}$ = 300

$\cos\theta = \frac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{OA \times AB} = \frac{(240)(240)+(70)(180)}{(250)(300)}$ (M1)

= 0.936 (A1)

$\Rightarrow \theta = 20.6°$ (A1) 4

(c) (i) $\overrightarrow{AX} = \begin{pmatrix} 339-240 \\ 238-70 \end{pmatrix} = \begin{pmatrix} 99 \\ 168 \end{pmatrix}$ (A1)

(ii) $\begin{pmatrix} -3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 240 \\ 180 \end{pmatrix}$ = –720 + 720 = 0 (M1)(A1)

$\Rightarrow \boldsymbol{n} \perp \overrightarrow{AB}$ (AG)

(iii) Projection of $\overrightarrow{AX}$ in the direction of $\boldsymbol{n}$ is

$$XY = \frac{1}{5}\begin{pmatrix} 99 \\ 168 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \frac{-297+672}{5} = 75$$ (M1)(A1)(A1) 6

(d) $AX = \sqrt{99^2 + 168^2} = 195$ (A1)

$AY = \sqrt{195^2 - 75^2} = 180$ km (M1)(A1) 3

**[18]**

**13.** (a) At $t = 2$, $\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0.7 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.4 \\ 2 \end{pmatrix}$ (M1)

Distance from (0, 0) = $\sqrt{3.4^2 + 2^2} = 3.94$ m (A1) 2

(b) $\left|\begin{pmatrix} 0.7 \\ 1 \end{pmatrix}\right| = \sqrt{0.7^2 + 1^2}$ (M1)

$= 1.22 \text{ m s}^{-1}$ (A1) 2

(c) $x = 2 + 0.7\,t$ and $y = t$ (M1)

$x - 0.7y = 2$ (A1) 2

(d) $y = 0.6x + 2$ and $x - 0.7y = 2$ (M1)

$x = 5.86$ and $y = 5.52$ $\left(\text{or } x = \frac{170}{29} \text{ and } y = \frac{160}{29}\right)$ (A1)(A1) 3

(e) The time of the collision may be found by solving

$\begin{pmatrix}5.86\\5.52\end{pmatrix}=\begin{pmatrix}2\\0\end{pmatrix}+\begin{pmatrix}0.7\\1\end{pmatrix}t$ for $t$ (M1)

$\Rightarrow t = 5.52$ s (A1)

[*ie* collision occurred 5.52 seconds after the vehicles set out].

Distance $d$ travelled by the motorcycle is given by

$d = \left|\begin{pmatrix}5.86\\5.52\end{pmatrix}-\begin{pmatrix}0\\2\end{pmatrix}\right| = \sqrt{(5.86)^2+(3.52)^2}$ (M1)

$= \sqrt{46.73}$

$= 6.84$ m (A1)

Speed of the motorcycle $= \dfrac{d}{t} = \dfrac{6.84}{5.52}$

$= 1.24$ m s$^{-1}$ (A1) 5

**[14]**

**14.** (a) $\boldsymbol{u} \bullet \boldsymbol{v} = 8 + 3 + p$ (A1)

For equating scalar product equal to zero (M1)

$8 + 3 + p = 0$

$p = -11$ A1 N3

(b) $|\boldsymbol{u}| = \sqrt{2^2+3^2+(-1)^2} = \sqrt{14}$, 3.74 (M1)

$q\sqrt{14} = 14$ A1

$q = \sqrt{14}$ $(= 3.74)$ A1 N2

**[6]**

**15.** (a) $\overrightarrow{OG} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ A2 2

(b) $\overrightarrow{BD} = 5\boldsymbol{i} + 5\boldsymbol{k}$ A2 2

(c) $\overrightarrow{EB} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ A2 2

***Note**: Award A0(A2)(A2) if the 5 is consistently omitted.*

**[6]**

**16.** (a) (i) evidence of combining vectors (M1)

*eg* $\vec{AB} = \vec{OB} - \vec{OA}$ (or $\vec{AD} = \vec{AO} + \vec{OD}$ in part (ii))

$\vec{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ A1 N2

(ii) $\vec{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ A1 N1

(b) evidence of using perpendicularity $\Rightarrow$ scalar product = 0 (M1)

$e.g. \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$

$4 - 4(k - 5) + 4 = 0$ A1

$-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) A1

$k = 7$ AG N0

(c) $\vec{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ (A1)

$\vec{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ A1

evidence of correct approach (M1)

*eg* $\vec{OC} = \vec{OB} + \vec{BC}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ A1 N3

(d) **METHOD 1**

choosing appropriate vectors, $\overrightarrow{BA}$, $\overrightarrow{BC}$ (A1)

finding the scalar product M1

*eg* $-2(1) + 4(1) + 2(-1)$, $2(1) + (-4)(1) + (-2)(-1)$

$\cos A\hat{B}C = 0$ A1 N1

**METHOD 2**

$\overrightarrow{BC}$ parallel to $\overrightarrow{AD}$ (may show this on a diagram with points labelled) R1

$\overrightarrow{BC} \perp \overrightarrow{AB}$ (may show this on a diagram with points labelled) R1

$A\hat{B}C = 90°$

$\cos A\hat{B}C = 0$ A1 N1

**[13]**

**17.** (a) Attempting to find unit vector ($e_b$) in the direction of $\boldsymbol{b}$ (M1)

Correct values = $\dfrac{1}{\sqrt{3^2 + 4^2 + 0^2}}\begin{pmatrix}3\\4\\0\end{pmatrix}$ A1

$= \begin{pmatrix}0.6\\0.8\\0\end{pmatrix}$ A1

Finding direction vector for $\boldsymbol{b}$, $\boldsymbol{v_b} = 18 \times \boldsymbol{e_b}$ (M1)

$\boldsymbol{b} = \begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$ A1

Using vector representation $\boldsymbol{b} = \boldsymbol{b}_0 + t\boldsymbol{v_b}$ (M1)

$= \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$ AG 6

| | | | | |
|---|---|---|---|---|
| (b) | (i) | $t = 0 \Rightarrow (49, 32, 0)$ | A1 | 1 |
| | (ii) | Finding magnitude of velocity vector | (M1) | |
| | | Substituting correctly $v_h = \sqrt{(-48)^2 + (-24)^2 + 6^2}$ | A1 | |
| | | $= 54(\text{km h}^{-1})$ | A1 | 3 |
| (c) | (i) | At R, $\begin{pmatrix} 10.8t \\ 14.4t \\ 5 \end{pmatrix} = \begin{pmatrix} 49 - 48t \\ 32 - 24t \\ 6t \end{pmatrix}$ | A1 | |
| | | $t = \frac{5}{6}$ (= 0.833) (hours) | A1 | 2 |
| | (ii) | For substituting $t = \frac{5}{6}$ into expression for $b$ or $h$ | M1 | |
| | | (9,12,5) | A2 | 3 |

**[15]**

**18.** (a) (i) Evidence of subtracting all three components in the correct order M1

*eg* $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (4\boldsymbol{i} - 5\boldsymbol{j} + 21\boldsymbol{k}) - (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k})$

$= 2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k}$ AG N0

(ii) $|\overrightarrow{AB}| = \sqrt{2^2 + (-8)^2 + 20^2} \; \left(= \sqrt{468} = 6\sqrt{13} = 2\sqrt{117} = 21.6\right)$ (A1)

$\boldsymbol{u} = \frac{1}{\sqrt{468}}(2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k})$ A1 N2

$\left(= \frac{2}{\sqrt{468}}\boldsymbol{i} - \frac{8}{\sqrt{468}}\boldsymbol{j} + \frac{20}{\sqrt{468}}\boldsymbol{k}, 0.0925\boldsymbol{i} - 0.370\boldsymbol{j} + 0.925\boldsymbol{k}, \text{etc.}\right)$

(iii) If the scalar product is zero, the vectors are perpendicular. R1

***Note**: Award R1 for stating the relationship between the scalar product and perpendicularity, seen anywhere in the solution.*

Finding an appropriate scalar product $\left(\boldsymbol{u} \bullet \overrightarrow{OA} \text{ or } \overrightarrow{AB} \bullet \overrightarrow{OA}\right)$ M1

*eg* $\boldsymbol{u} \bullet \overrightarrow{OA} = \left(\frac{2}{\sqrt{468}}\right) \times 2 + \left(\frac{-8}{\sqrt{468}}\right) \times 3 + \left(\frac{20}{\sqrt{468}}\right) \times 1$

$\left(= \frac{4 - 24 + 20}{\sqrt{468}}\right)$

$\overrightarrow{AB} \bullet \overrightarrow{OA} = 2 \times 2 + (-8) \times 3 + 20 \times 1$

$\boldsymbol{u} \bullet \overrightarrow{OA} = 0$ or $\overrightarrow{AB} \bullet \overrightarrow{OA} = 0$ A1 N0

(b) (i) **EITHER**

$S\left(\frac{2+4}{2}, \frac{3-5}{2}, \frac{1+21}{2}\right)$ (M1)(A1)

Therefore, $\vec{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ (accept (3, –1, 11)) A1 N3

**OR**

$\vec{OS} = \vec{OA} + \frac{1}{2}\vec{AB}$ (M1)

$= (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}) + \frac{1}{2}(2\boldsymbol{i} + 8\boldsymbol{j} + 20\boldsymbol{k})$ (A1)

$\vec{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ A1 N3

(ii) $L_1 : r = (3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}) + t\,(2\boldsymbol{i} + 3\boldsymbol{j} + 1\boldsymbol{k})$ A1 N1

(c) Using direction vectors (*eg* $2\boldsymbol{i} + 3\boldsymbol{j} + 1\boldsymbol{k}$ and $-2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k}$) (M1)
Valid explanation of why $L_1$ is not parallel to $L_2$ R1 N2

*eg.* Direction vectors are not scalar multiples of each other.
Angle between the direction vectors is not zero or 180.
Finding the angle

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 \neq |\boldsymbol{d}_1||\boldsymbol{d}_2|$.

***Note**: Award R0 for "direction vectors are not equal".*

(d) Setting up any **two** of the three equations (M1)
For each correct equation A1A1
*eg* $3 + 2t = 5 - 2s$, $-1 + 3t = 10 + 5s$, $11 + t = 10 - 3s$

Attempt to solve these equations (M1)
Finding **one** correct parameter ($s = -1$, $t = 2$) (A1)

P has position vector $7\boldsymbol{i} + 5\boldsymbol{j} + 13\boldsymbol{k}$ A2 N4

***Notes**: Award (M1)A2 if the same parameter is used for both lines in the initial correct equations. Award no further marks.*

**[19]**