IB MATHEMATICS SL

VECTORS WORKSHEET #2

1. A boat B moves with constant velocity along a straight line. Its velocity vector is given by

 $\boldsymbol{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$

At time t = 0 it is at the point (-2, 1).

- (a) Find the magnitude of v.
- (b) Find the coordinates of B when t = 2.
- (c) Write down a vector equation representing the position of B, giving your answer in the form r = a + tb.

Working:	
	Answers:
	(a)
	(b)
	(c)
	(Total 6 marks)

- **2.** A triangle has its vertices at A(-1, 3), B(3, 6) and C(-4, 4).
 - (a) Show that $\overrightarrow{AB} \bullet \overrightarrow{AC} = -9$
 - (b) Show that, to three significant figures, $\cos BAC = -0.569$.

- 3. (a) Find the scalar product of the vectors $\begin{pmatrix} 60\\25 \end{pmatrix}$ and $\begin{pmatrix} -30\\40 \end{pmatrix}$.
 - (b) Two markers are at the points P (60, 25) and Q (-30, 40). A surveyor stands at O (0, 0) and looks at marker P. Find the angle she turns through to look at marker Q.

Working:	
	Answers:
	(a)
	(b)
	(Total 6 marks)

4. The point O has coordinates (0, 0, 0), point A has coordinates (1, -2, 3) and point B has coordinates (-3, 4, 2).

(a) (i) Show that
$$\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$$
.

(b) The line
$$L_1$$
 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1 .

(2)

(6)

(8)

- (c) The line L_2 passes through A and is parallel to \overrightarrow{OB} .
 - (i) Find a vector equation for L_2 , giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.
 - (ii) Point C (k, -k, -5) is on L_2 . Find the coordinates of C.

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of *p* at C.

(2) (Total 18 marks) 5. The diagram below shows a cuboid (rectangular solid) OJKLMNPQ. The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).



(ii) Find \overrightarrow{MK} .

(2)

(9)

(b) An equation for the line (MK) is
$$\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$$
.

(i) Write down an equation for the line (JQ) in the form r = a + tb.

(ii) Find the acute angle between (JQ) and (MK).

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D.

(5) (Total 16 marks) 6. In this question, distance is in metres, time is in minutes.

Two model airplanes are each flying in a straight line.

At 13:00 the first model airplane is at the point (3, 2, 7). Its position vector after t minutes is

given by
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$$
.

(a) Find the speed of the model airplane.

(2)

At 13:00 the second model airplane is at the point (-5, 10, 23). After two minutes, it is at the point (3, 16, 39).

- (b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$. (3)
- (c) The airplanes meet at point Q.
 - (i) At what time do the airplanes meet?
 - (ii) Find the position of Q.

(6)

(d) Find the angle θ between the paths of the two airplanes.

(6) (Total 17 marks)

- 7. Consider the point D with coordinates (4, 5), and the point E, with coordinates (12, 11).
 - (a) Find \overrightarrow{DE} . (2)
 - (b) Find $|\overrightarrow{DE}|$.

(2)

(c) The point D is the centre of a circle and E is on the circumference as shown in the following diagram.



The point G is also on the circumference. \overrightarrow{DE} is perpendicular to \overrightarrow{DG} . Find the possible coordinates of G.

(8) (Total 12 marks) 8. In this question the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents a displacement of 1 km east, and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents a displacement of 1 km north.

The diagram below shows the positions of towns A, B and C in relation to an airport O, which is at the point (0, 0). An aircraft flies over the three towns at a constant speed of 250 km h^{-1} .



Town A is 600 km west and 200 km south of the airport. Town B is 200 km east and 400 km north of the airport. Town C is 1200 km east and 350 km south of the airport.

(a)

(i) Find \overrightarrow{AB} .

(ii) Show that the vector of length one unit in the direction of \overrightarrow{AB} is $\begin{pmatrix} 0.8\\ 0.6 \end{pmatrix}$.

An aircraft flies over town A at 12:00, heading towards town B at 250 km h^{-1} .

Let $\begin{pmatrix} p \\ q \end{pmatrix}$ be the velocity vector of the aircraft. Let *t* be the number of hours in flight after 12:00. The position of the aircraft can be given by the vector equation

$$\binom{x}{y} = \binom{-600}{-200} + t\binom{p}{q}.$$

(b) (i) Show that the velocity vector is $\begin{pmatrix} 200\\ 150 \end{pmatrix}$.

- (ii) Find the position of the aircraft at 13:00.
- (iii) At what time is the aircraft flying over town B?

(4)

Over town B the aircraft changes direction so it now flies towards town C. It takes five hours to travel the 1250 km between B and C. Over town A the pilot noted that she had 17 000 litres of fuel left. The aircraft uses 1800 litres of fuel per hour when travelling at 250 km h^{-1} . When the fuel gets below 1000 litres a warning light comes on.

(c) How far from town C will the aircraft be when the warning light comes on?

(7) (Total 17 marks)