

IB STANDARD LEVEL MATHEMATICS

ANALYZING FUNCTIONS

4 Answer the whole of this question on a sheet of graph paper.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-8	4.5	8	5.5	0	-5.5	-8	-4.5	8

- (a) Using a scale of 2 cm to represent 1 unit on the x -axis and 2 cm to represent 4 units on the y -axis, draw axes for $-4 \leq x \leq 4$ and $-8 \leq y \leq 8$.
Draw the curve $y = f(x)$ using the table of values given above. [5]
- (b) Use your graph to solve the equation $f(x) = 0$. [2]
- (c) On the same grid, draw $y = g(x)$ for $-4 \leq x \leq 4$, where $g(x) = x + 1$. [2]
- (d) Write down the value of
- (i) $g(1)$,
 - (ii) $fg(1)$,
 - (iii) $g^{-1}(4)$,
 - (iv) the **positive** solution of $f(x) = g(x)$. [4]
- (e) Draw the tangent to $y = f(x)$ at $x = 3$. Use it to calculate an estimate of the gradient of the curve at this point. [3]
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Answer all of this question on a sheet of graph paper.

(a) $f(x) = x^2 - x - 3$.

x	-3	-2	-1	0	1	2	3	4
$f(x)$	p	3	-1	-3	q	-1	3	r

- (i) Find the values of p , q and r . [3]
- (ii) Draw the graph of $y = f(x)$ for $-3 \leq x \leq 4$.
Use a scale of 1 cm to represent 1 unit on each axis. [4]
- (iii) By drawing a suitable line, estimate the gradient of the graph at the point where $x = -1$. [3]

(b) $g(x) = 6 - \frac{x^3}{3}$.

x	-2	-1	0	1	2	3
$g(x)$	8.67	u	v	5.67	3.33	-3

- (i) Find the values of u and v . [2]
- (ii) On the same grid as **part (a) (ii)** draw the graph of $y = g(x)$ for $-2 \leq x \leq 3$. [4]
- (c) (i) Show that the equation $f(x) = g(x)$ simplifies to $x^3 + 3x^2 - 3x - 27 = 0$. [1]
- (ii) Use your graph to write down a solution of the equation $x^3 + 3x^2 - 3x - 27 = 0$. [1]

Answer the whole of this question on a sheet of graph paper.

The table gives values of $f(x) = 2^x$, for $-2 \leq x \leq 4$.

x	-2	-1	0	1	2	3	4
$f(x)$	p	0.5	q	2	4	r	16

- (a) Find the values of p , q and r . [3]
- (b) Using a scale of 2 cm to 1 unit on the x -axis and 1 cm to 1 unit on the y -axis, draw the graph of $y = f(x)$ for $-2 \leq x \leq 4$. [5]
- (c) Use your graph to solve the equation $2^x = 7$. [1]
- (d) What value does $f(x)$ approach as x decreases? [1]
- (e) By drawing a tangent, estimate the gradient of the graph of $y = f(x)$ when $x = 1.5$. [3]
- (f) On the same grid draw the graph of $y = 2x + 1$ for $0 \leq x \leq 4$. [2]
- (g) Use your graph to find the non-integer solution of $2^x = 2x + 1$. [2]

Answer only **one** of the following two alternatives.

EITHER

The equation of a curve is $y = 2x^2 - 20x + 37$.

(i) Express y in the form $a(x + b)^2 + c$, where a , b and c are integers. [3]

(ii) Write down the coordinates of the stationary point on the curve. [1]

A function f is defined by $f : x \mapsto 2x^2 - 20x + 37$ for $x > k$. Given that the function $f^{-1}(x)$ exists,

(iii) write down the least possible value of k , [1]

(iv) sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes provided, [2]

(v) obtain an expression for f^{-1} . [3]

OR

A function g is defined by $g : x \mapsto 5x^2 + px + 72$, where p is a constant. The function can also be written as $g : x \mapsto 5(x - 4)^2 + q$.

(i) Find the value of p and of q . [3]

(ii) Find the range of the function g . [1]

(iii) Sketch the graph of the function on the axes provided. [2]

(iv) Given that the function h is defined by $h : x \mapsto \ln x$, where $x > 0$, solve the equation $gh(x) = 12$. [4]

Answers

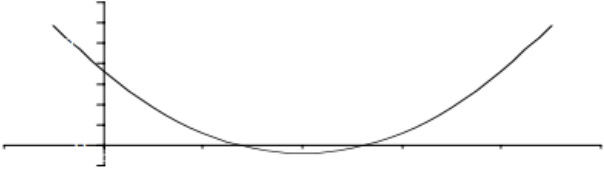
4	(a)	Scales correct 9 points correctly plotted (1mm) Reasonable curve through 9 points	S1 P3 C1√ (5)	$-4 \leq x \leq 4$ and $-8 \leq y \leq 8$ Allow P2 for 7 or 8 correct, P1 for 5 or 6 correct √ provided shape maintained, curvature OK and <u>not</u> ruled
	(b)	$-3.6 \leq x \leq -3.3$, $x = 0$, $3.3 \leq x \leq 3.6$	B2 (2)	Allow B1 for 1 correct non-zero solution; condone (-3.5, 0) (answers must be in range <u>and</u> correct for their graph)
	(c)	Line from (-4, -3) to (4, 5), and ruled	B2 (2)	If B0, allow B1 for gradient 1 or intercept 1 on single line
	(d)	$g(1) = 2$ $fg(1) = -8$ $g^{-1}(4) = 3$ $3.75 \leq x \leq 3.9$	B1 B1 B1 B1 (4)	Not (1, 2) Lost if y-coordinate given. Answer must be OK for their graph

	(e)	Tangent drawn at $x = 3$ on curve Vert./Horiz. using scale Answer in range 5-10 and OK for theirs	B1 M1 A1 (3)	Not chord or daylight Dep. on reasonable approx to tangent used at $x = 3$ (N.B. Gradient = 4.5 + y-value of tangent at $x = 4$)
TOTAL			16	
5	(a)	$\frac{1}{2} 10 \cdot 10 \cdot \sin 60^\circ$ o.e.	M1	Any complete method including $\sqrt{15.5.5.5}$

Q2(a)(i)	$p = 9$ $q = -3$ $r = 9$	1+1+1	Must be seen. No feedback from graph.
(ii)	Scales correct Their 8 points plotted correctly (1mm) Reasonable curve through all 8 of their points (1mm tolerance)	S1 √ P2 √	x from -3 to 4. y to accommodate their values. P1 √ for 6 or 7 of their points correct. Condone ruled line for $x = 3$ to 4 or -3 to -2.
(iii)	Tangent drawn at $x = -1$ on curve -3.5 to -2.5 Condone fractions	C1 √ T1 B2	ft provided correct shape maintained. Or a parallel line drawn. If B2 not scored, give B1 for 2.5 to 3.5 after M1.
(b)(i)	$u = 6.33$ or better $v = 6$	1+1	Allow $u = 19/3$
(ii)	Their 6 points plotted correctly (1mm) Reasonable curve through all 6 of their points (1mm tolerance)	P3 √ C1 √	P2 for 5 correct (√). P1 for 4 correct (√). Condone ruled line for $x = 2$ to 3. ft provided correct shape maintained
(c)(i)	$x^2 - x - 3 = 6 - x^3/3$ o.e. to $x^3 + 3x^2 - 3x - 27 = 0$	E1	At least 1 intermediate step and no errors seen.
(ii)	2.3 to 2.7 c.a.o.	B1	<u>Not</u> coordinates
Q3(a)(i)	Median 36 to 37 (cm)	B1	18

4 (a)	$p = 0.25$ $q = 1$ $r = 8$	1* B1 B1 B1	Must be seen. No feedback from graph. If not labelled, must be in order
(b)	Scales correct Their 7 points plotted correctly (within 1mm and in the correct square) Smooth curve through all 7 points (1mm)	S1 P3 ✓ C1 ✓	x from -2 to 4 . y to accommodate their values. ft P2 for 6 points correct. P1 for 5 points correct. ft provided correct shape maintained
(c)	2.75 to 2.85	B1	
(d)	0	B1	
(e)	Tangent drawn at $x = 1.5$ Uses <u>increase in y</u> (using scale) <u>increase in x</u> 1.7 to 2.2	T1 M1 A1	Not a chord and no daylight Dep on T1 or a near miss (not chord or clearly drawn at $x = 1$ or $x = 2$) If correct method seen, condone any answer in range, even with a slight slip
(f)	Correct ruled straight line (complete for range 0 to 4)	B2	SC1 for freehand complete line or any ruled line of gradient 2 or y -intercept of 1 (<u>not</u> $y=1$)
(g)	Correct for theirs (± 0.05) dep. on at least SC1 in (f)	B2 ✓	SC1 if y -coordinate also given or $x=0$ also given (or both)
		17	

Page 8	Mark Scheme: Teachers' version	Syllabus	Paper
	IGCSE – May/June 2012	0606	21

120 (i)	$5(x^2 - 8x + 16) + q$ and equate $p = -40$ $q = -8$	M1 A1 A1
(ii)	$g(x) > -8$	B1 ✓
(iii)	 <p>positive quadratic curve correct position</p>	B1 B1 ✓
(iv)	$5(\ln x - 4)^2 - 8 = 12$ solve for $(\ln x - 4)$ $\ln x = 2, 6$ $x = 7.39$ or e^2 , $x = 403$ or e^6	B1 M1 A1 A1
	$5(\ln x)^2 - 40 \ln x + 60 = 0$ solve 3 term quadratic $\ln x = 2, 6$ $x = 7.39$ or e^2 , $x = 403$ or e^6	
		[10]