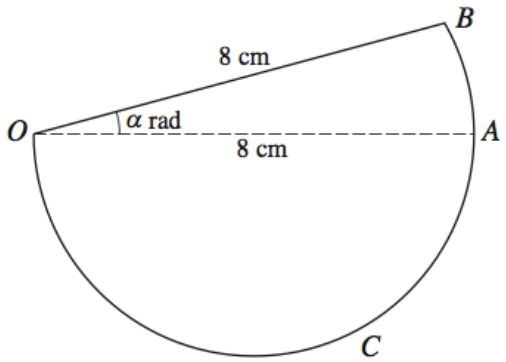
TRIGONOMETRY – ARCS AND SECTORS WORKSHEET 1 QSNS 1 - 6

1.

CIE A LEVEL PAPER 11 JUNE 2013 - QUESTION 3



In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB.

(i) Find α in terms of π. [3]

(ii) Find the perimeter of the complete figure in terms of π. [2]

(i) $(OAB)=\frac{1}{2}\times 8^2\alpha$, $(OAC)=\frac{1}{2}\times\pi\times 4^2$ **B1B1**

$\alpha=\frac{\pi}{8}$ **B1**

[3]

(ii) $8+8\times$ their $\alpha+\frac{1}{2}\times 8\times\pi$ **B1** √

$8+5\pi$ **B1**

[2]

2.

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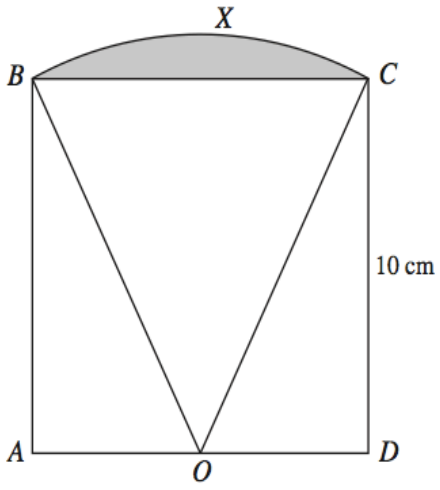

The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O.

(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]

(i) $BOC = 2\tan^{-1}\frac{1}{2} = 0.9273$	M1 A1 [2]
(ii) $OB = \sqrt{(10^2 + 5^2)}$ or $11.2 = r$ Arc $BXC = \sqrt{125} \times 0.9273$ $\rightarrow$ Perimeter = 20.4 cm	B1 M1 A1 [3]
(iii) Area $= \frac{1}{2}r^2\theta$ $-\frac{1}{2}.10.10 \rightarrow 7.96$ cm^2.	M1 A1 [2]

3.

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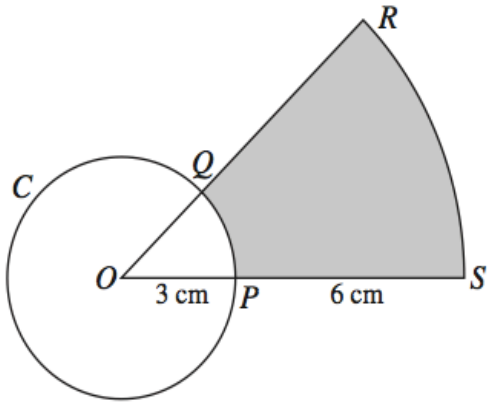

The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that $PS = 6$ cm and that the area of the shaded region is equal to the area of circle C,

(i) show that angle $POQ = \frac{1}{4}\pi$ radians, [3]

(ii) find the perimeter of the shaded region. [2]

(i)	$\frac{1}{2}.3^2\pi = \frac{1}{2}9^2\theta - \frac{1}{2}3^2\theta$	M1 A1
	$\rightarrow \theta = \frac{1}{4}\pi$	A1
(ii)	$P = 6+6+3\times\frac{1}{4}\pi + 9\times\frac{1}{4}\pi = 21.4$ cm.	M1
	or $12 + 3\pi$	A1

4.

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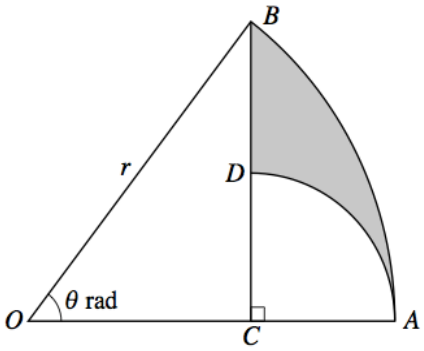

The diagram shows a sector OAB of a circle with centre O and radius r. Angle AOB is θ radians. The point C on OA is such that BC is perpendicular to OA. The point D is on BC and the circular arc AD has centre C.

(i) Find AC in terms of r and θ. [1]

(ii) Find the perimeter of the shaded region ABD when $\theta = \frac{1}{3}\pi$ and $r = 4$, giving your answer as an exact value. [6]

(i) $AC = r - r\cos\theta$ **B1** [1]

(ii) arc $AB = \frac{4\pi}{3}$ **B1**

arc $AD = \frac{\pi}{2} \times their\ AC = \frac{\pi}{2} \times \left(4 - 4\cos\frac{\pi}{3}\right) = \pi$ **M1A1**

$BD = 4\sin\frac{\pi}{3} - their\ AC = 2\sqrt{3} - 2$ **M1A1**

Perimeter $= \frac{7\pi}{3} + 2\sqrt{3} - 2$ **A1** [6]

5.

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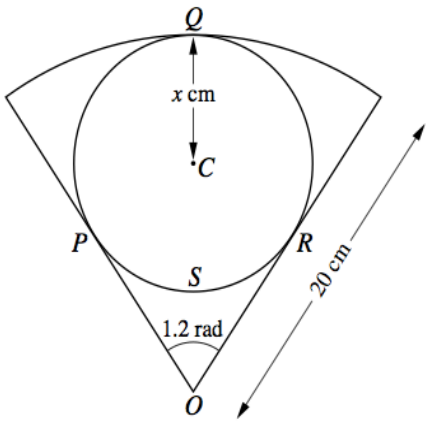

The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P, Q and R. Angle POR = 1.2 radians.

(i) Show that $x = 7.218$, correct to 3 decimal places. [4]

(ii) Find the total area of the three parts of the sector lying outside the circle with centre C. [2]

(iii) Find the perimeter of the region $OPSR$ bounded by the arc PSR and the lines OP and OR. [4]

(i) $OQ = x + OC = 20$ — B1

$\sin 0.6 = \dfrac{x}{OC} \rightarrow OC = \dfrac{x}{\sin 0.6}$ — M1

$x + \dfrac{x}{\sin 0.6} = 20 \rightarrow x = 7.218$ — M1 A1 [4]

(ii) Area $= ½ . 20^2 \times 1.2 - \pi \times 7.218^2$ — M1
$= 76.3$ — A1 [2]

(iii) Angle $PCR = \pi - 1.2$ — B1
Arc $PR = 7.218 \times (\pi - 1.2) = (14.01)$ — M1

$OP = OR = \dfrac{x}{\tan 0.6}$ — M1

$\rightarrow$ Perimeter of 35.1 cm — A1 [4]

6.

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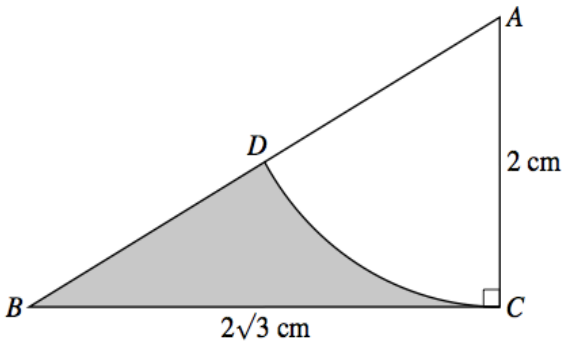

In the diagram, D lies on the side AB of triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. [4]

area $\Delta = 2\sqrt{3}$	B1
$\tan A = \frac{2\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{3}$	B1
Area sector $= \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3}$	M1
Shaded area	A1
	[4]