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MATHEMATICS

Standard Level

The portfolio – tasks

For use in 2011 and 2012

For final assessment in 2011 and 2012

CONTENTS

Introduction

What is the purpose of this document?

This document contains new tasks for the portfolio in mathematics SL. These tasks have been produced by the IB, for teachers to use in 2011 and 2012. It should be noted that any tasks previously produced and published by the IB will no longer be valid for assessment after the November 2010 examination session. These include all the tasks in any teacher support material (TSM), and the tasks in the document “Portfolio tasks 2009–2010”. Copies of all TSM tasks published by the IB are available on the Online Curriculum Centre (OCC), under **Internal Assessment**, in a document called “Old tasks published prior to 2008”. These tasks should **not** be used, even in slightly modified form.

What happens if teachers use these old tasks?

The inclusion of these old tasks in the portfolio will make the portfolio non-compliant, and such portfolios will therefore attract a 10-mark penalty. Teachers may continue to use the old tasks as practice tasks, but they should not be included in the portfolio for final assessment.

What other documents should I use?

All teachers should have copies of the mathematics SL subject guide (second edition, September 2006), including the teaching notes appendix, and the TSM (September 2005). Further information, including additional notes on applying the criteria, is available on the Online Curriculum Centre (OCC). Important news items are also available on the OCC, as are the diploma programme coordinator notes, which contain updated information on a variety of issues.

Which tasks can I use in 2011?

The only tasks produced by the IB that may be submitted for assessment in 2011 are the ones contained in this document. (These tasks may also be used in 2012, along with the other tasks for 2012 and 2013 that will be published by January 2011.) There is no requirement to use tasks produced by the IB, and there is no date restriction on tasks written by teachers.

Can I use these tasks before May 2011?

These tasks should only be submitted for final assessment from May 2011 to November 2012. Students should not include them in portfolios before May 2011. If they are included, they will be subject to a 10-mark penalty. Please note that these dates refer to examination sessions, not when the work is completed.

INFINITE SUMMATION **SL TYPE I**

Aim: In this task, you will investigate the sum of infinite sequences t_n, where

$$t_0 = 1,\ t_1 = \frac{(x\ln a)}{1},\ t_2 = \frac{(x\ln a)^2}{2\times 1},\ t_3 = \frac{(x\ln a)^3}{3\times 2\times 1} \ldots,\ t_n = \frac{(x\ln a)^n}{n!} \ldots.$$

A notation that you may find helpful in this task is the factorial notation $n!$, defined by

$$n! = n(n-1)(n-2)....3\times 2\times 1 \qquad e.g.\ 5! = 5\times 4\times 3\times 2\times 1\ (=120) \qquad \text{Note that } 0! = 1$$

Consider the following sequence of terms where $x = 1$ and $a = 2$.

$$1,\ \frac{(\ln 2)}{1},\ \frac{(\ln 2)^2}{2\times 1},\ \frac{(\ln 2)^3}{3\times 2\times 1} \ldots$$

Calculate the sum S_n of the first n terms of the above sequence for $0 \le n \le 10$. Give your answers correct to six decimal places.

Using technology, plot the relation between S_n and n. Describe what you notice from your plot. What does this suggest about the value of S_n as n approaches ∞?

Consider another sequence of terms where $x = 1$ and $a = 3$.

$$1,\ \frac{(\ln 3)}{1},\ \frac{(\ln 3)^2}{2\times 1},\ \frac{(\ln 3)^3}{3\times 2\times 1} \ldots$$

Calculate the sum S_n of the first n terms of this new sequence for $0 \le n \le 10$. Give your answers correct to six decimal places.

Using technology, plot the relation between S_n and n. Describe what you notice from your plot. What does this suggest about the value of S_n as n approaches ∞?

Now consider a general sequence where $x = 1$.

$$1,\ \frac{(\ln a)}{1},\ \frac{(\ln a)^2}{2\times 1},\ \frac{(\ln a)^3}{3\times 2\times 1} \ldots$$

Calculate the sum S_n of the first n terms of this general sequence for $0 \le n \le 10$ for different values of a. Give your answers correct to six decimal places.

Using technology, plot the relation between S_n and n. Describe what you notice from your plot. What does this suggest about the value of S_n as n approaches ∞?

Use your observations from these investigations to find a general statement that represents the infinite sum of this general sequence.

(This task continues on the following page)

Now we will expand our investigation to determine the sum of the infinite sequence t_n, where

$$t_0 = 1,\ t_1 = \frac{(x\ln a)}{1},\ t_2 = \frac{(x\ln a)^2}{2\times 1},\ t_3 = \frac{(x\ln a)^3}{3\times 2\times 1}\ \ldots.$$

Define $T_n(a,\ x)$ as the sum of the first n terms, for various values of a and x,
e.g. $T_9(2,\ 5)$ is the sum of the first nine terms when $a = 2$ and $x = 5$.

Let $a = 2$. Calculate $T_9(2,\ x)$ for various positive values of x. Using technology, plot the relation between $T_9(2,\ x)$ and x. Describe what you notice from your plot.

Let $a = 3$. Calculate $T_9(3,\ x)$ for various positive values of x. Using technology, plot the relation between $T_9(3,\ x)$ and x. Describe what you notice from your plot.

Continue with this analysis to find the general statement for $T_n(a,\ x)$ as n approaches ∞.

Test the validity of the general statement with other values of a and x.

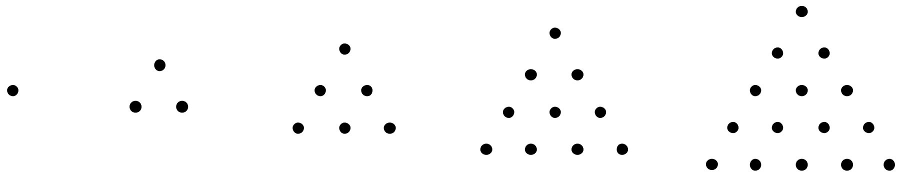
Discuss the scope and/or limitations of the general statement.

Explain how you arrived at the general statement.

STELLAR NUMBERS **SL TYPE I**

Aim: In this task you will consider geometric shapes which lead to special numbers. The simplest example of these are ***square*** *numbers, 1, 4, 9, 16, which can be represented by squares of side 1, 2, 3 and 4.*

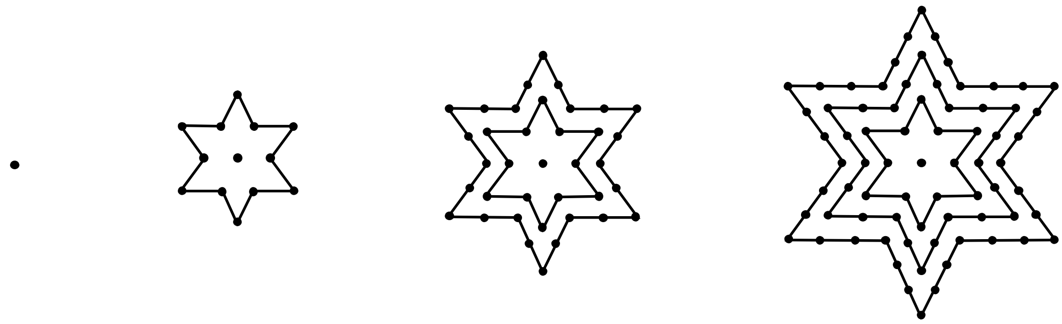
The following diagrams show a triangular pattern of evenly spaced dots. The numbers of dots in each diagram are examples of **triangular** numbers $(1, 3, 6, \ldots)$.



1 3 6 10 15

Complete the triangular numbers sequence with three more terms.
Find a general statement that represents the n^{th} triangular number in terms of n.

Consider **stellar** (star) shapes with p vertices, leading to p-stellar numbers. The first four representations for a star with six vertices are shown in the four stages S_1–S_4 below. The 6-stellar number at each stage is the total number of dots in the diagram.



S_1 S_2 S_3 S_4

Find the number of dots (*i.e.* the stellar number) in each stage up to S_6. Organize the data so that you can recognize and describe any patterns.

Find an expression for the 6-stellar number at stage S_7.

Find a general statement for the 6-stellar number at stage S_n in terms of n.

Now repeat the steps above for other values of p.

Hence, produce the general statement, in terms of p and n, that generates the sequence of p-stellar numbers for any value of p at stage S_n.

Test the validity of the general statement.

Discuss the scope or limitations of the general statement.

Explain how you arrived at the general statement.

POPULATION TRENDS IN CHINA **SL TYPE II**

Aim: In this task, you will investigate different functions that best model the population of China from 1950 to 1995.

The following table[1] shows the population of China from 1950 to 1995.

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Population in Millions	554.8	609.0	657.5	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

Define all relevant variables and parameters clearly. Use technology to plot the data points from the above table on a graph.

Comment on any apparent trends shown in the graph. What types of functions could model the behaviour of the graph? Explain your choices.

Analytically develop **one** model function that fits the data points on your graph.

On a new set of axes, plot your model and the original data. Comment on how well your model fits the original data. Revise your model if necessary.

A researcher suggests that the population, P at time t can be modelled by

$$P(t) = \frac{K}{1 + Le^{-Mt}}, \text{ where } K, L \text{ and } M \text{ are parameters.}$$

Use technology to estimate and interpret K, L and M. Construct the researcher's model using your estimates.

On a new set of axes, plot the researcher's model and the original data. Comment on how well this model fits the original data.

Discuss the implications of each of these models in terms of population growth for China in the future.

Here are additional data on population trends in China from the 2008 World Economic Outlook, published by the International Monetary Fund (IMF).

Year	1983	1992	1997	2000	2003	2005	2008
Population in Millions	1030.1	1171.7	1236.3	1267.4	1292.3	1307.6	1327.7

Comment on how well each of the models above fit the IMF data for the years 1983–2008.

Modify the model that best fits the IMF data, so that it applies to all the given data from 1950 to 2008. Comment on how well your modified model fits all the data.

[1] Data from the "land use change and agriculture program", published by the *International Institute for Applied Systems Analysis*

G-FORCE TOLERANCE **SL TYPE II**

Aim: In this task you will develop model functions representing the tolerance of human beings to G-forces over time.

Introduction and background

When different forces are applied to an object, "G-force" is a term used to describe the resulting acceleration, and is in relation to acceleration due to gravity (g). Thus a G-force equivalent to twice the force of gravity is 2g ("2 gees").

Astronauts have developed their own terminology, based on sensations. Forward acceleration is often referred to as "eyeballs-in", where the G-force pushes the body backwards, while backward acceleration is referred to as "eyeballs-out". Similarly, vertical acceleration upwards is referred to as "blood towards feet".

In general, humans have a greater tolerance to forward acceleration than backward acceleration, since blood vessels in the retina appear more sensitive in the latter direction. Early experiments showed that untrained humans were able to tolerate 17g eyeballs-in (compared to 12g eyeballs-out) for several minutes without loss of consciousness or apparent long-term harm. The human body is also considerably better at surviving G-forces that are horizontal, that is, perpendicular to the spine.

Amusement park rides such as roller coasters subject humans to a variety of forces in different directions. These rides typically do not expose humans to much more than about 3g of horizontal force for approximately three seconds. There are exceptions which reach a maximum of 4.5g for up to 1.3 seconds.

The task

The following table illustrates the tolerance of human beings to **horizontal** G-force. The notation "+Gx" represents a positive acceleration in the horizontal direction *i.e.* eyeballs-in, so that a force of +Gx of 20 means a forward acceleration of 20 (which humans can tolerate for 0.1 minutes).

Time (min)	**+Gx (g)**
0.01	35
0.03	28
0.1	20
0.3	15
1	11
3	9
10	6
30	4.5

Define appropriate variables and parameters, and identify any constraints for the data.

Using technology plot the data points on a graph. Comment on any apparent trends shown in the graph.

(This task continues on the following page)

What type of function models the behaviour of the graph? Explain why you chose this function. Create an equation (a model) that fits the graph.

On a new set of axes, draw your model function and the original data points. Comment on any differences. Revise your model if necessary. Discuss the implications of your model in terms of G-forces acting on a human being.

Use technology to find another function that models the data. On a new set of axes, draw your model function and the function you found using technology. Comment on any differences.

The table below illustrates the tolerance of human beings to **vertical** G-forces. The notation "+Gz" represents a positive acceleration in the vertical direction *i.e.* blood towards feet.

Time (min)	**+Gz (g)**
0.01	18
0.03	14
0.1	11
0.3	9
1	7
3	6
10	4.5
30	3.5

How well does your first model fit this new data?

What changes, if any, need to be made to your model to fit this new data?

Discuss any limitations to your model and the implications of your model in terms of G-forces acting on a human being.