## MATHEMATICS

## Standard Level

The portfolio - tasks

## For use in 2012 and 2013

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## INTRODUCTION

## What is the purpose of this document?

This document contains new tasks for the portfolio in mathematics SL. These tasks have been produced by the IB, for teachers to use in the examination sessions in 2012 and 2013. It should be noted that most tasks previously produced and published by the IB will no longer be valid for assessment after the November 2010 examination session. These include all the tasks in any teacher support material (TSM), and the tasks in the document "Portfolio tasks 2009-2010". The tasks in the document "Portfolio tasks 2011-2012" can be used in the 2012 examinations but NOT in 2013.

Copies of all TSM tasks published by the IB are available on the Online Curriculum Centre (OCC), under Internal Assessment, in a document called "Old tasks published prior to 2008". These tasks should not be used, even in slightly modified form.

## What happens if teachers use these old tasks?

The inclusion of these old tasks in the portfolio will make the portfolio non-compliant, and such portfolios will therefore attract a 10 -mark penalty. Teachers may continue to use the old tasks as practice tasks, but they should not be included in the portfolio for final assessment.

## What other documents should I use?

All teachers should have copies of the mathematics SL subject guide (second edition, September 2006), including the teaching notes appendix, and the TSM (September 2005). Further information, including additional notes on applying the criteria, is available on the Online Curriculum Centre (OCC). Important news items are also available on the OCC, as are the diploma programme coordinator notes, which contain updated information on a variety of issues.

## Which tasks can I use in 2012?

The only tasks produced by the IB that may be submitted for assessment in 2012 are the ones contained in this document, and those in the document "Portfolio tasks 2011-2012". There is no requirement to use tasks produced by the IB, and there is no date restriction on tasks written by teachers.

## Can I use these tasks before May 2012?

These tasks should only be submitted for final assessment from May 2012 to November 2013. Students should not include them in portfolios before May 2012. If they are included, they will be subject to a 10 -mark penalty. Please note that these dates refer to examination sessions, not when the work is completed.

## Which tasks can I use in 2013?

The only tasks produced by the IB that may be submitted for assessment in 2013 are the ones contained in this document.

## Technology

There is a wide range of technological tools available to support mathematical work. These include graphic display calculators, Excel spreadsheets, Geogebra, Autograph, Geometer sketch pad and Wolframalpha. Many are free downloads from the Internet. Students (and teachers) should be encouraged to explore which ones best support the tasks that are assigned. Teachers are reminded that good technology use should enhance the development of the task.

## Extracts from diploma program coordinator notes

Important information is included in the DPCN, available on the OCC. Teachers should ensure they are familiar with these, and in particular with the ones noted below. Please note that the reference to the 2009/2010 document is outdated.

## Copies of tasks and marking/solution keys

Teachers are advised to write their own tasks to fit in with their own teaching plans, to select from the 2009/2010 document, or to use tasks written by other teachers. In each case, teachers should work the task themselves to make sure it is suitable, and provide a copy of the task, and an answer, solution or marking key for any task submitted. This will help the moderators confirm the levels awarded by the teacher.

It is particularly important if teachers modify an IB published task to include a copy of the modified task. While this is permitted, teachers should think carefully about making any changes, as the tasks have been written with all the criteria in mind, to allow students to achieve the higher levels.

## Non-compliant portfolios from May 2012

Please note the following information on how to deal with portfolios that do not contain one task of each type. This will be applied in the May 2012 and subsequent examination sessions.

If two pieces of work are submitted, but they do not represent a Type I and a Type II task (for example, they are both Type I or both Type II tasks), mark both tasks, one against each Type.
For example, if a candidate has submitted two Type I tasks, mark one using the Type I criteria, and the other using the Type II Criteria. Do not apply any further penalty

This means that the current system of marking both tasks against the same criteria and then applying a penalty of 10 marks will no longer be used.

## LACSAP'S FRACTIONS

Aim: In this task you will consider a set of numbers that are presented in a symmetrical pattern. Consider the five rows of numbers shown below.

$$
\begin{aligned}
& 1 \quad 1 \\
& 1 \quad \frac{3}{2} \quad 1 \\
& 1 \quad \frac{6}{4} \quad \frac{6}{4} \quad 1 \\
& \begin{array}{lllll}
1 & \frac{10}{7} & \frac{10}{6} & \frac{10}{7} & 1
\end{array} \\
& 1 \quad \frac{15}{11} \quad \frac{15}{9} \quad \frac{15}{9} \quad \frac{15}{11} \quad 1
\end{aligned}
$$

Describe how to find the numerator of the sixth row.
Using technology, plot the relation between the row number, $n$, and the numerator in each row. Describe what you notice from your plot and write a general statement to represent this.

Find the sixth and seventh rows. Describe any patterns you used.
Let $E_{n}(r)$ be the $(r+1)^{\text {th }}$ element in the $n^{\text {th }}$ row, starting with $r=0$.
Example: $E_{5}(2)=\frac{15}{9}$.

Find the general statement for $E_{n}(r)$.
Test the validity of the general statement by finding additional rows.
Discuss the scope and/or limitations of the general statement.
Explain how you arrived at your general statement.

## CIRCLES

Aim: The aim of this task is to investigate positions of points in intersecting circles.
The following diagram shows a circle $C_{1}$ with centre O and radius $r$, and any point P .


The circle $C_{2}$ has centre P and radius OP. Let A be one of the points of intersection of $C_{1}$ and $C_{2}$. Circle $C_{3}$ has centre A , and radius $r$. The point $\mathrm{P}^{\prime}$ is the intersection of $C_{3}$ with (OP). This is shown in the diagram below.


Let $r=1$. Use an analytic approach to find $\mathrm{OP}^{\prime}$, when $\mathrm{OP}=2$, $\mathrm{OP}=3$ and $\mathrm{OP}=4$. Describe what you notice and write a general statement to represent this.

Let $\mathrm{OP}=2$. Find $\mathrm{OP}^{\prime}$, when $r=2, r=3$ and $r=4$. Describe what you notice and write a general statement to represent this. Comment whether or not this statement is consistent with your earlier statement

Use technology to investigate other values of $r$ and OP. Find the general statement for $\mathrm{OP}^{\prime}$.
Test the validity of your general statement by using different values of OP and $r$.
Discuss the scope and/or limitations of the general statement.
Explain how you arrived at the general statement.

## FISH PRODUCTION

Aim: This task considers commercial fishing in a particular country in two different environments the sea and fish farms (aquaculture). The data is taken from the UN Statistics Division Common Database.

The following table gives the total mass of fish caught in the sea, in thousands of tonnes ( 1 tonne $=1000$ kilograms ).

| Year | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Mass | 426.8 | 470.2 | 503.4 | 557.3 | 564.7 | 575.4 | 579.8 | 624.7 | 669.9 |


| Year | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Mass | 450.5 | 379.0 | 356.9 | 447.5 | 548.8 | 589.8 | 634.0 | 527.8 | 459.1 |


| Year | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Mass | 487.2 | 573.8 | 503.3 | 527.7 | 566.7 | 507.8 | 550.5 | 426.5 | 533.0 |

Define suitable variables and discuss any parameters/constraints. Using technology, plot the data points from the table on a graph. Comment on any apparent trends in your graph and suggest suitable models.

Analytically develop a model that fits the data points. (You may find it useful to consider a combination of functions.)

On a new set of axes, draw your model function and the original data points. Comment on any differences. Revise your model if necessary.

The table below gives the total mass of fish, in thousands of tonnes, from fish farms.

| Year | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Mass | 1.4 | 1.5 | 1.7 | 2.0 | 2.2 | 2.7 | 3.1 | 3.3 | 4.1 |


| Year | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Mass | 4.4 | 5.8 | 7.8 | 9.1 | 12.4 | 16.0 | 21.6 | 33.2 | 45.5 |


| Year | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Mass | 56.7 | 63.0 | 79.0 | 67.2 | 61.2 | 79.9 | 94.7 | 119.8 | 129.0 |

Plot the data points from this table on a graph, and discuss whether your analytical model for the original data fits the new data.

Use technology to find a suitable model for the new data. On a new set of axes, draw both models.
Discuss how trends in the first model could be explained by trends in the second model.
By considering both models, discuss possible future trends in both types of fishing.

## GOLD MEDAL HEIGHTS

Aim: The aim of this task is to consider the winning height for the men's high jump in the Olympic Games.

The table below gives the height (in centimeters) achieved by the gold medalists at various Olympic Games.

| Year | 1932 | 1936 | 1948 | 1952 | 1956 | 1960 | 1964 | 1968 | 1972 | 1976 | 1980 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height (cm) | 197 | 203 | 198 | 204 | 212 | 216 | 218 | 224 | 223 | 225 | 236 |

Note: The Olympic Games were not held in 1940 and 1944.
Using technology, plot the data points on a graph. Define all variables used and state any parameters clearly. Discuss any possible constraints of the task.

What type of function models the behaviour of the graph? Explain why you chose this function. Analytically create an equation to model the data in the above table.

On a new set of axes, draw your model function and the original graph. Comment on any differences. Discuss the limitations of your model. Refine your model if necessary.

Use technology to find another function that models the data. On a new set of axes, draw both your model functions. Comment on any differences.

Had the Games been held in 1940 and 1944, estimate what the winning heights would have been and justify your answers.

Use your model to predict the winning height in 1984 and in 2016. Comment on your answers.
The following table gives the winning heights for all the other Olympic Games since 1896.

| Year | 1896 | 1904 | 1908 | 1912 | 1920 | 1928 | 1984 | 1988 | 1992 | 1996 | 2000 | 2004 | 2008 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height (cm) | 190 | 180 | 191 | 193 | 193 | 194 | 235 | 238 | 234 | 239 | 235 | 236 | 236 |

How well does your model fit the additional data?
Discuss the overall trend from 1896 to 2008, with specific references to significant fluctuations.
What modifications, if any, need to be made to your model to fit the new data?

