The difference between these times is 1 hour:

$$\frac{180}{x} - \frac{180}{x+15} = 1$$

Multiply across by x and by $(x + 15)$, then rearrange and solve:

$$180(x + 15) - 180x = x(x + 15)$$
$$180x + 2700 - 180x = x^2 + 15x$$
$$x^2 + 15x - 2700 = 0$$
$$(x - 45)(x + 60) = 0$$

So $x = 45$ or $x = -60$. Ignore the negative answer.

Her speed was 45 km/h.

Exercise 2.10

1 The length of a rectangular sheet of paper is 3 cm greater than the width. If the area is 70 cm^2, find the length.

2 The area of a circle is 35π greater than its perimeter. Find the radius.

3 Two numbers differ by 7. Their product is 60. Find the numbers.

4 The sides of a right-angled triangle are $(2x + 1)$ cm, $2x$ cm and $(x - 1)$ cm. Find x.

5 A ball is thrown up in the air and, after t seconds, its height, h metres, is given by $h = 30t - 5t^2$. Find when the ball is 40 m high.

6 A man can swim at x m/s. He swims in a river where the current is 2 m/s. He swims against the current for 600 m and then with the current for 600 m. The total time is 225 seconds. Find x.

7 A plane can fly at x km/h. There is a wind of 20 km/h. A journey of 960 km against the wind takes 2 hours longer than the same journey with the wind. Find x.

8 A stone is thrown upwards. After t seconds its height, h metres, is given by $h = 30t - 5t^2$. Find the times when the stone is 25 m high.

9 A stone is thrown downwards from the top of a tower. After t seconds it has fallen s m, where $s = 10t + 5t^2$. Find when it has fallen 50 m.

10 A man walks x km north then $(x + 10)$ km east. He is now 100 km from his starting point. Find x.

11 The width of a rectangle is x cm, and its length is $(x + 2)$ cm. Find the value of x for which the area of the rectangle is 27 cm^2.

12 The length of a rectangle is 2 m greater than the width. The area is 27 m^2. Let the width be x m. Find an equation in x and solve it.

13 The two sides of a rectangle are x cm and $(x - 2)$ cm. The diagonal of the rectangle is 20 cm. Find x.

14 The shorter sides of a right-angled triangle are x cm and $(x + 3)$ cm. The area of the triangle is 4 cm^2. Find x.

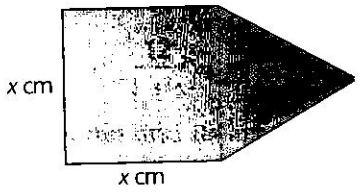
15 The hypotenuse of a right-angled triangle is $(x + 8)$ cm and the two shorter sides are $(x + 2)$ cm and $(x - 3)$ cm. Find x.

16 The area of a hall is 300 m^2. If the width is decreased by 1 m and the length increased by 2 m the area is unchanged. Find the original width.

17 If a cylinder has height h and radius r, then its surface area is $2\pi r^2 + 2\pi rh$. Find the radius of a cylinder with height 8 cm and surface area 100π cm^2.

18 A cylinder has height 10 cm and surface area 20 cm^2. Find its radius.

19 The shape shown is a square of side x cm with an isosceles triangle at one end. If the total area is 40 cm^2, find x.



20 The diagram shows a race track, with two straight parts and two semicircles at the ends. The straight parts are $2x$ m apart and they each have length $(2x + 30)$ m. If the area enclosed by the track is 20 000 m^2, find x.

21 A bus regularly goes on a journey of 60 km. If the speed of the bus is increased by 3 km per hour, the journey will take 0.1 hour less. Find the original speed of the bus.

22 Mr Jones walks on a journey of 30 km and back. His average speed for the return journey was 1 km/h greater than for the outward journey. The total time was 9 hours. Find his speed on the outward journey.

23 The current in a river is 2 km per hour. A man can row at x km per hour. He rows 15 km with the current, then 15 km against the current. The total time taken is 8 hours. Find x.

24 The average age of a class is 15 years 2 months. A new student aged 16 years 2 months joins and the average is now 15 years 3 months. Find how many students there are in the class.

25 An amount of gas has mass 1.2 kg. If its volume increases by 10 m^3, then its density decreases by 0.01 kg/m^3. Find its original volume.

2.11 Simultaneous equations: one linear, one quadratic

In Book 1 we solved simultaneous linear equations. If one of the equations is quadratic, we may be able to solve it by the method of substitution.

Example 19

Solve the equations $y + 2x = 3$ and $x^2 + y^2 = 2$.

Solution

Use the first equation to write y in terms of x:

$$y = 3 - 2x$$

Substitute into the second equation:

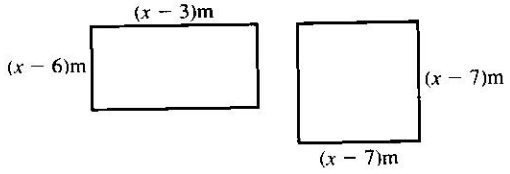
$$x^2 + (3 - 2x)^2 = 2$$
$$x^2 + 9 - 12x + 4x^2 = 2$$
$$5x^2 - 12x + 7 = 0$$

Exercise 30

Solve by forming a quadratic equation:

Questions 4, 6 and 7 use Pythagoras' Theorem.

1. Two numbers, which differ by 3, have a product of 88. Find them.
2. The product of two consecutive odd numbers is 143. Find the numbers. (Hint: If the first odd number is x, what is the next odd number?)
3. The length of a rectangle exceeds the width by 7 cm. If the area is 60 cm^2, find the length of the rectangle.
4. The length of a rectangle exceeds the width by 2 cm. If the diagonal is 10 cm long, find the width of the rectangle.
5. The area of the rectangle exceeds the area of the square by 24 m^2. Find x.



6. The perimeter of a rectangle is 68 cm. If the diagonal is 26 cm, find the dimensions of the rectangle.
7. A man walks a certain distance due North and then the same distance plus a further 7 km due East. If the final distance from the starting point is 17 km, find the distances he walks North and East.
8. A farmer makes a profit of x cents on each of the $(x + 5)$ eggs her hen lays. If her total profit was 84 cents, find the number of eggs the hen lays.
9. A boy buys x eggs at $(x - 8)$ cents each and $(x - 2)$ rashers of bacon at $(x - 3)$ cents each. If the total bill is \$1·75, how many eggs does he buy?
10. A number exceeds four times its reciprocal by 3. Find the number.
11. Two numbers differ by 3. The sum of their reciprocals is $\frac{7}{10}$; find the numbers.
12. A cyclist travels 40 km at a speed of x km/h. Find the time taken in terms of x. Find the time taken when his speed is reduced by 2 km/h. If the difference between the times is 1 hour, find the original speed x.
13. An increase of speed of 4 km/h on a journey of 32 km reduces the time taken by 4 hours. Find the original speed.
14. A train normally travels 240 km at a certain speed. One day, due to bad weather, the train's speed is reduced by 20 km/h so that the journey takes two hours longer. Find the normal speed.

Exercise 2.10

1 10 cm
3 5 and 12
5 2 and 4 seconds
7 140
9 2.32 s
11 4.8
13 15.1
15 20.7
17 4.12
19 6.12
21 41 km/h
23 4.62
25 30 m^3

page 76 ***Exercise 30***

1. 8, 11 **2.** 11, 13 **3.** 12 cm **4.** 6 cm

5. $x = 11$ **6.** 10 cm × 24 cm **7.** 8 km north, 15 km east **8.** 12 eggs

9. 13 eggs **10.** 4 or -1 **11.** 2, 5 **12.** $\frac{40}{x}$ h, $\frac{40}{x-2}$ h, 10 km/h

13. 4 km/h **14.** 60 km/h **15.** 5 km/h **16.** 157 km **17.** $x = 2$

18. $x = 3$ or 9·5 **19.** $\frac{3}{4}$ **20.** 9 cm or 13 cm