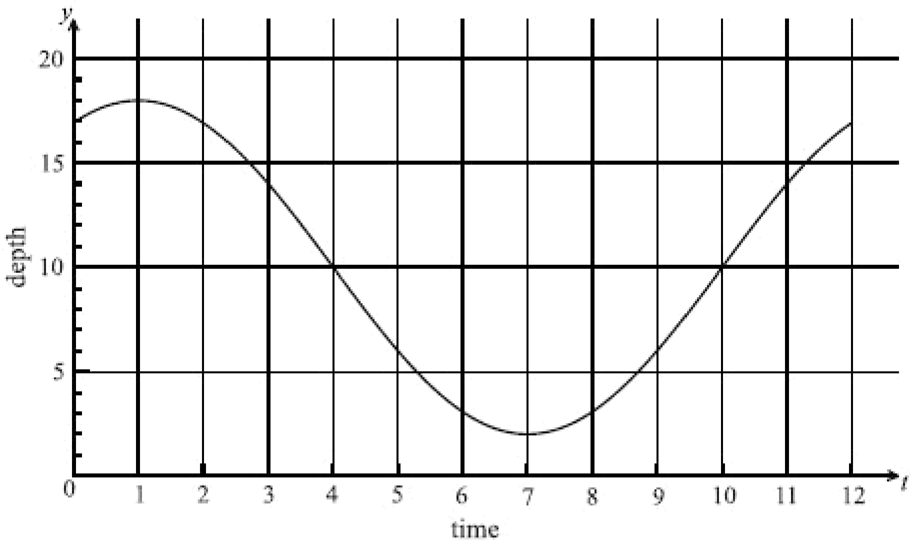
IB MATHEMATICS SL

Topic: TRIGONOMETRY

1. The following graph shows the depth of water, *y* metres, at a point P, during one day. The time *t* is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of *t* when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

(3)

(b) The depth of water can be modelled by the function $y = A \cos (B (t - 1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B.

(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of *t* between which he cannot sail past P.

(2)
(Total 11 marks)

2. The diagram below shows a circle centre O, with radius r. The length of arc ABC is 3π cm and $A\hat{O}C = \frac{2\pi}{9}$.

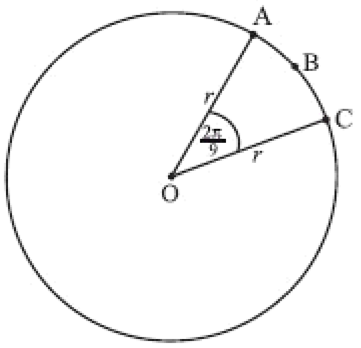


diagram not to scale

(a) Find the value of r.

(2)

(b) Find the perimeter of sector OABC.

(2)

(c) Find the area of sector OABC.

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(2)
(Total 6 marks)

3. The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $P\hat{Q}R$ is 75°.

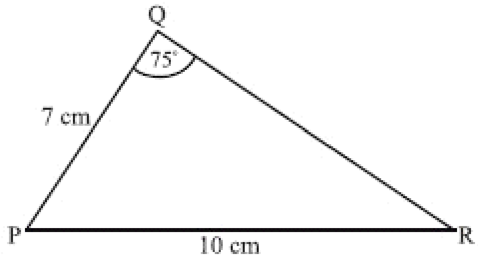

diagram not to scale

(a) Find $P\hat{Q}R$.

(3)

(b) Find the area of triangle PQR.

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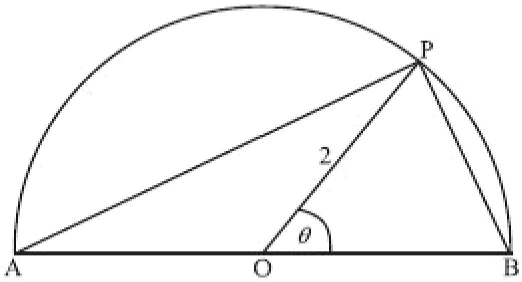
(3)
(Total 6 marks)

4. The following diagram shows a semicircle centre O, diameter [AB], with radius 2.
Let P be a point on the circumference, with $\hat{POB} = \theta$ radians.



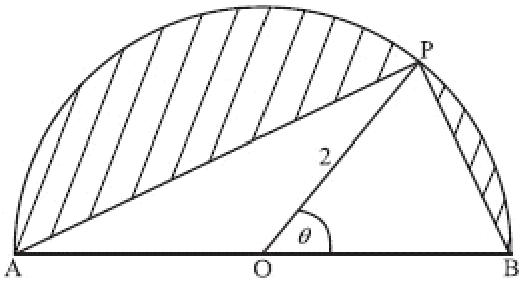
(a) Find the area of the triangle OPB, in terms of θ.

(2)

(b) Explain why the area of triangle OPA is the same as the area triangle OPB.

(3)

Let S be the total area of the two segments shaded in the diagram below.



(c) Show that $S = 2(\pi - 2 \sin \theta)$.

(3)

(d) Find the value of θ when S is a local minimum, justifying that it is a minimum.

(8)

(e) Find a value of θ for which S has its greatest value.

(2)
(Total 18 marks)

5. (a) Given that $\cos A = \frac{1}{3}$ and $0 \leq A \leq \frac{\pi}{2}$, find $\cos 2A$.

(3)

(b) Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \leq B \leq \pi$, find $\cos B$.

(3)

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(Total 6 marks)

6. Consider $g(x) = 3 \sin 2x$.

(a) Write down the period of g.

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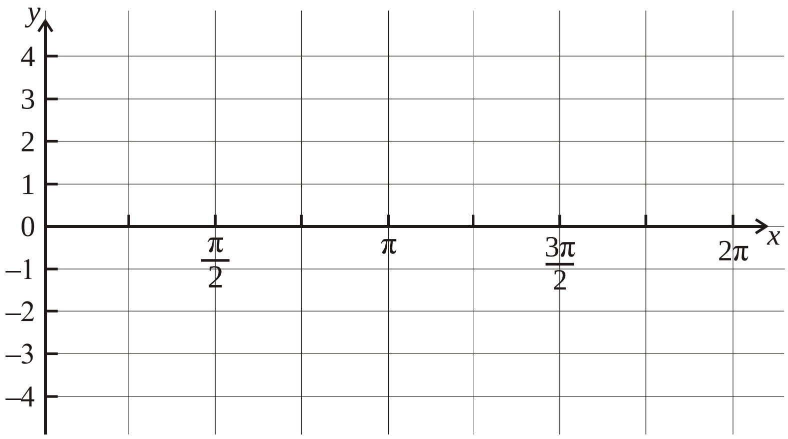
(1)

(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$.



(3)

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$.

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(2)
(Total 6 marks)

7. Let $p = \sin 40°$ and $q = \cos 110°$. Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 140°$;

(ii) $\cos 70°$.

(2)

(b) Find an expression for $\cos 140°$.

(3)

(c) Find an expression for $\tan 140°$.

(1)

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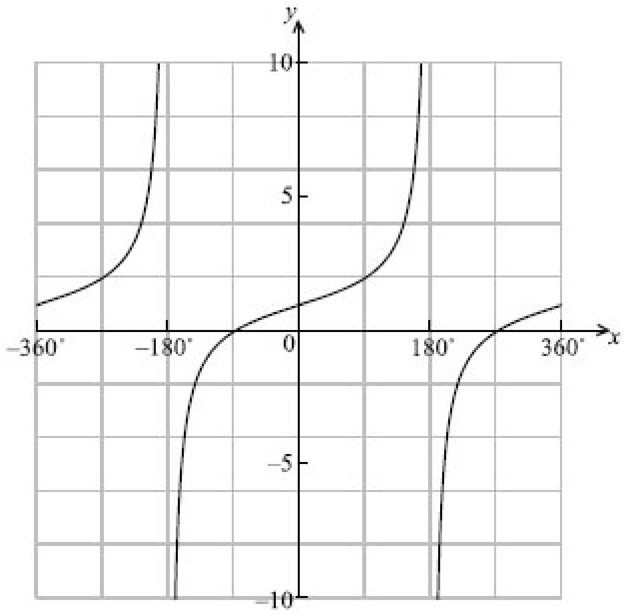
(Total 6 marks)

8. The diagram below shows the graph of $f(x) = 1 + \tan\left(\frac{x}{2}\right)$ for $-360° \le x \le 360°$.



(a) On the same diagram, draw the asymptotes.

(2)

(b) Write down

(i) the period of the function;

(ii) the value of $f(90°)$.

(2)

(c) Solve $f(x) = 0$ for $-360° \le x \le 360°$.

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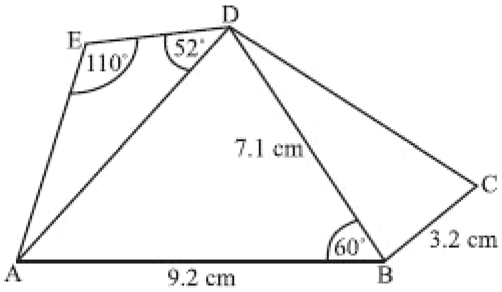
(2)
(Total 6 marks)

9. The following diagram shows a pentagon ABCDE, with AB = 9.2 cm, BC = 3.2 cm, BD = 7.1 cm, $A\hat{E}D$ =110°, $A\hat{D}E$ = 52° and $A\hat{B}D$ = 60°.



(a) Find AD.

(4)

(b) Find DE.

(4)

(c) The area of triangle BCD is 5.68 cm^2. Find $D\hat{B}C$.

(4)

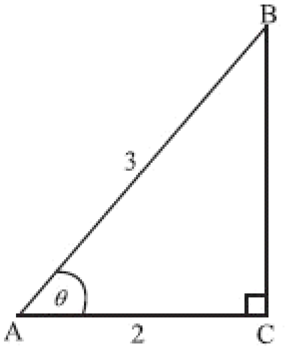
(d) Find AC.

(4)

(e) Find the area of quadrilateral ABCD.

(5)
(Total 21 marks)

10. The following diagram shows a triangle ABC, where $A\hat{C}B$ is 90°, AB = 3, AC = 2 and $B\hat{A}C$ is θ.



(a) Show that $\sin\theta = \dfrac{\sqrt{5}}{3}$.

(b) Show that $\sin 2\theta = \dfrac{4\sqrt{5}}{9}$.

(c) Find the **exact** value of $\cos 2\theta$.

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(Total 6 marks)

11. The function f is defined by $f : x \rightarrow 30 \sin 3x \cos 3x$, $0 \le x \le \dfrac{\pi}{3}$.

(a) Write down an expression for $f(x)$ in the form $a \sin 6x$, where a is an integer.

(b) Solve $f(x) = 0$, giving your answers in terms of π.

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(Total 6 marks)

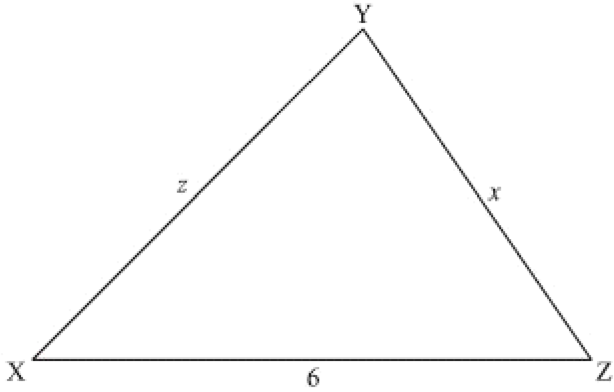
12. (a) Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

(i) the value of x giving the maximum value of y;

(ii) this maximum value of y.

The triangle XYZ has XZ = 6, YZ = x, XY = z as shown below. The perimeter of triangle XYZ is 16.

(4)



(b) (i) Express z in terms of x.

(ii) Using the cosine rule, express z^2 in terms of x and cos Z.

(iii) Hence, show that $\cos Z = \dfrac{5x-16}{3x}$.

(7)

Let the area of triangle XYZ be A.

(c) Show that $A^2 = 9x^2 \sin^2 Z$.

(2)

(d) Hence, show that $A^2 = -16x^2 + 160x - 256$.

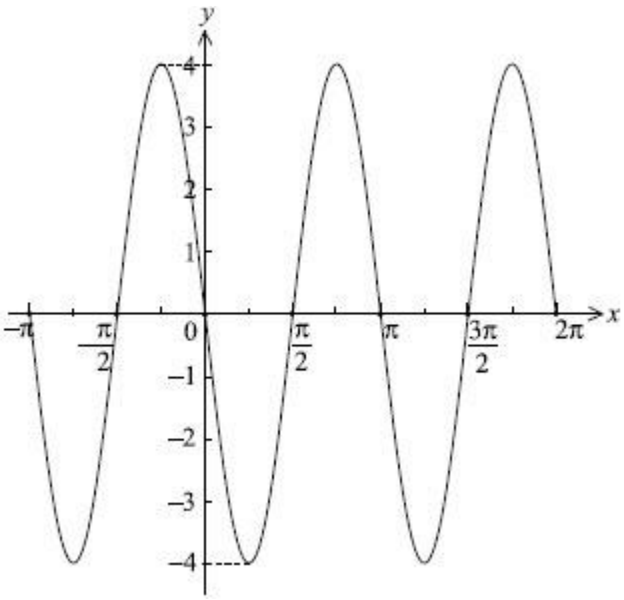
(4)

(e) (i) Hence, write down the maximum area for triangle XYZ.

(ii) What type of triangle is the triangle with maximum area?

(3)
(Total 20 marks)

13. Let $f(x) = a \sin b(x - c)$. Part of the graph of f is given below.



Given that a, b and c are positive, find the value of a, of b and of c.

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(Total 6 marks)

14. Consider the equation $3 \cos 2x + \sin x = 1$

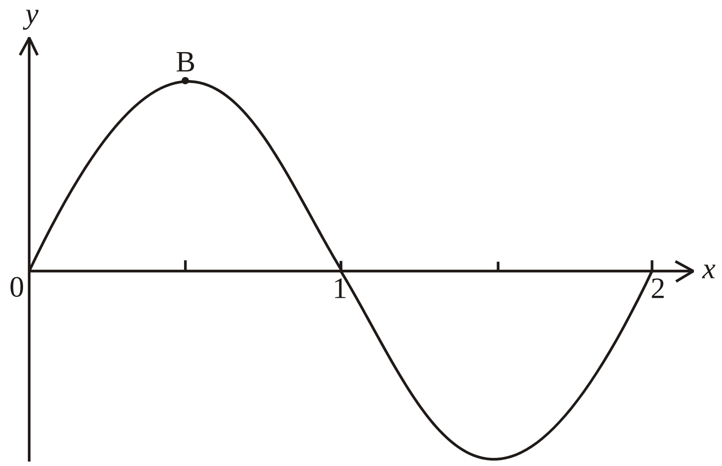
(a) Write this equation in the form $f(x) = 0$, where $f(x) = p \sin^2 x + q \sin x + r$, and $p, q, r \in \mathbb{Z}$.

(b) Factorize $f(x)$.

(c) Write down the number of solutions of $f(x) = 0$, for $0 \le x \le 2\pi$.

(Total 6 marks)

15. Let $f(x) = 6 \sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of f is shown on the diagram below. There is a maximum value at B (0.5, b).



(a) Write down the value of b.

(b) On the same diagram, sketch the graph of g.

(c) Solve $f(x) = g(x)$, $0.5 \le x \le 1.5$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)